Federal state autonomous educational institution of higher education

«Peoples' Friendship University of Russia »

Faculty of science

STATE FINAL CERTIFICATION PROGRAM
for students in the programs of
training of scientific and pedagogical personnel in postgraduate studies
Training direction
01.06.01 Mathematics and Mechanics

Profile:
Real, complex, and functional analysis

(higher education – training of highly qualified personnel)

Graduate qualification (degree):
The researcher. Research teacher

1. GENERAL PROVISIONS

1.1. The procedure for conducting the final state certification of students determines the responsibility and procedure for the preparation and conduct of the state final tests in the RUDN, as well as the list, order, and deadlines for passing the documents required for the implementation of the state final certification.

1.2. The state final certification in the direction of training of scientific and pedagogical personnel in the postgraduate school 01.06.01 "Mathematics and Mechanics" in the profile "Real, complex and functional analysis" includes the state exam corresponding to the candidate's minimum program in the specialty "Real, complex and functional analysis", and the defense of the scientific report on the main results of the scientific qualification work (dissertation).

1.3. The results of any of the types of attestation included in the state final examination is determined by the grade "excellent", "good", "satisfactory", "unsatisfactory".

1.4. The state final certification is carried out according to the "Regulation on the conduct of state final certification under educational programs of higher education — programs of preparation of scientific and pedagogical personnel in postgraduate study, approved by Order of the rector of 20.01.2017, No. 41.

1.5. Preparation and registration of scientific and qualification work (dissertation) is carried out in accordance with the" Regulations for the preparation and registration of scientific and qualification work (dissertation) on the programs of training of highly qualified personnel at the Peoples 'Friendship University of Russia", approved by the Rector's Order No. 40 of 20.01.017, and GOST R 7.0.11-2011 " Dissertation and abstract of the dissertation. Structure and rules of registration".

2. GOALS AND OBJECTIVES OF THE STATE FINAL CERTIFICATION

2.1. The purpose of the state final certification is to determine the compliance of the results of mastering the main educational programs by students with the requirements of the HEP RUDN for postgraduate programs in the direction of training 01.06.01 "Mathematics and Mechanics" in the profile "Real, complex and functional analysis" (qualification: Researcher. Teacher-researcher). The state final certification includes a state examination established by the Academic Council of the university, and the presentation of a scientific report on the topic of the dissertation research.

2.2. The task of the state final certification is to check the level of formation of competencies defined by the HEP the RUDN and the training direction " Material, complex and functional analysis»:

universal competencies

− ability to critically analyze and evaluate modern scientific achievements, generate new ideas in solving research and practical problems, including in interdisciplinary areas (UC-1);

− the ability to design and implement comprehensive research, including interdisciplinary research, based on a holistic, systematic scientific worldview, using knowledge in the field of history and philosophy of science (UC-2);

− willingness to participate in the work of Russian and international research teams to solve scientific and scientific-educational tasks (UC-3);

- readiness to use modern methods and technologies of scientific communication in the state and foreign languages, including readiness to communicate orally and in writing in Russian and foreign languages to solve the tasks of professional activity, possession of foreign language communicative competence in the official business, educational and professional, scientific, socio-cultural, everyday and everyday spheres of foreign language communication (UC-4);
- ability to follow ethical standards in professional activities (UC-5);

general professional competencies:

- knowledge of the methodology of theoretical and experimental research in the field of professional activity (GPC-1);
- knowledge of the culture of scientific research, including the use of modern information and communication technologies (GPC-2);

professional competencies:

- knowledge in the main sections of real, complex and functional analysis, including measure theory, Lebesgue integral theory, Fourier series, Fourier transform, theory of analytic functions, theory of metric, Banach and Hilbert spaces, theory of bounded and completely continuous operators in Banach spaces, theory of self-adjoint operators, spectral theory of operators (PC-1);
- knowledge of the theory of functional spaces (lebegova space, Sobolev space, Nikolsky-Besov space, etc.) and its applications to the theory of partial differential equations, including the solvability and smoothness of solutions of boundary value problems for elliptic equations, in the field of nonlinear analysis, the theory of extreme problems and optimal control (PC-2);
- the ability to formulate the task of the study and the ways of its implementation, to summarize the results and draw appropriate conclusions, must understand the practical aspects of the theoretical results (PC-3).

3. State Exam Program

3.1. The state examination is conducted orally with the use of examination tickets.

3.2. Within the framework of the state exam, the degree of development of the following competencies by students is checked:

universal competencies

- ability to critically analyze and evaluate modern scientific achievements, generate new ideas in solving research and practical problems, including in interdisciplinary areas (UC-1);

- the ability to design and implement comprehensive research, including interdisciplinary research, based on a holistic, systematic scientific worldview, using knowledge in the field of history and philosophy of science (UC-2);

- willingness to participate in the work of Russian and international research teams to solve scientific and scientific-educational tasks (UC-3);

- readiness to use modern methods and technologies of scientific communication in the state and foreign languages, including readiness to communicate orally and in writing in Russian and foreign languages to solve the tasks of professional activity, possession of foreign language communicative competence in the official business, educational and professional, scientific, socio-cultural, everyday and everyday spheres of foreign language communication (UC-4);
- ability to follow ethical standards in professional activities (UC-5);

general professional competencies:

- knowledge of the methodology of theoretical and experimental research in the field of professional activity (GPC-1);
- knowledge of the culture of scientific research, including the use of modern information and communication technologies (GPC-2);

professional competencies:

- knowledge in the main sections of real, complex and functional analysis, including measure theory, Lebesgue integral theory, Fourier series, Fourier transform, theory of analytic functions, theory of metric, Banach and Hilbert spaces, theory of bounded and completely continuous operators in Banach spaces, theory of self-adjoint operators, spectral theory of operators (PC-1);
- knowledge of the theory of functional spaces (lebegova space, Sobolev space, Nikolsky-Besov space, etc.) and its applications to the theory of partial differential equations, including the solvability and smoothness of solutions of boundary value problems for elliptic equations, in the field of nonlinear analysis, the theory of extreme problems and optimal control (PC-2);
- the ability to formulate the task of the study and the ways of its implementation, to summarize the results and draw appropriate conclusions, must understand the practical aspects of the theoretical results (PC-3).

3.3. The scope of the state exam.

The total number of examination tickets is determined by the number of students allowed to pass the state final certification in the relevant field of training. Number of questions in the exam ticket: 3.

By the decision of the examination commission, the student may be asked additional questions related to the main sections of the state exam program and included in the list of questions for preparing for the state exam.

3.4. Content of the State exam

The exam ticket must contain:

3 questions from the main part of the candidate's minimum program in the specialty " Real, complex and functional analysis»;

1. Real analysis

Measures, measurable functions, integral. Additive functions of sets (measures), countable additivity of measures. Construction of the Lebegov continuation. Measurable functions. Convergence of functions in measure and almost everywhere. Theorems of Yegorov and Luzin. The Lebesgue integral. The limit transition under the sign of the integral. Comparison of Lebesgue and Riemann integrals. Direct products of measures. Fubini's theorem.

The indefinite Lebesgue integral and the theory of differentiation. Differentiability of a monotone function almost everywhere. Functions with limited variation. The derivative of the indefinite Lebesgue integral. The problem of restoring a function from its derivative. Absolutely continuous functions. The Radon-Nicodemus theorem. The Stiltjes integral.

Spaces of summable functions and orthogonal series. Helder and Minkowski inequalities. Lp spaces, their completeness. Complete and closed systems of functions. Orthonormal systems in L2 and Parseval equality. Series over orthogonal systems; the tendency to zero of the Fourier coefficients of the summable function in the case of a uniformly bounded orthonormal system.

Trigonometric series. The Fourier transform. The convergence condition of the Fourier series. Representation of functions by singular integrals. Uniqueness of the decomposition of a function into a trigonometric series. Fourier transform of integrable and quadratically integrable functions. The uniqueness property for the Fourier transform. Plancherel's theorem. The Laplace transform. The Fourier-Stieltjes transform.

Smooth manifolds and differential forms. The tangent space to the manifold at a point. Differential forms on a manifold. External differential. Integral of the form over the manifold. The Stokes formula. Basic integral formulas of analysis.

2. Comprehensive analysis

Integral representations of analytic functions. Cauchy's integral theorem and its inversion (Morera's theorem). The Cauchy integral formula. The mean theorem. The maximum modulus principle. Schwartz's Lemma. Cauchy-type integral, its limit values. Sokhotsky formulas.

Series of analytical functions. Special points. Deductions. Uniformly convergent series of analytic functions; Weierstrass ' theorem. Representation of analytic functions by power series, Cauchy inequalities. Zeros of analytical functions. The uniqueness theorem. Isolated singular points (of an unambiguous character). Cauchy's deduction theorem. Calculation of integrals using deductions. The principle of the argument. Rouchet's theorem. Approximation of analytic functions by polynomials.

Integer and meromorphic functions. Growth of the whole function. Order and type. Weierstrass ' theorem on integer functions with given zeros; decomposition of an integer function into an infinite product. The case of finite-order integer functions, Hadamard's theorem. The Mittag—Leffler theorem on meromorphic functions with given poles and principal parts.

Conformal mappings. Conformal maps implemented by elementary functions. The principle of preserving the area. Single-leaf criteria. Riemann's theorem. Boundary correspondence theorems for conformal maps.

Analytical continuation. Analytic continuation and complete analytic function (in the sense of Weierstrass). The concept of a Riemann surface. Continuation along the curve. The monodromy theorem. Isolated singular points of analytic functions, branching points of infinite order. The principle of symmetry. The Christoffel-Schwartz formula. Modular function. Normal families of functions, the normality criterion. Picard's theorem.

Harmonic functions. Harmonic functions, their relation to analytical functions. Invariance of harmony under conformal substitution of variables. Infinite differentiability. The mean theorem and the maximum principle. The uniqueness theorem. The Dirichlet problem. The Poisson formula for a circle.

3. Functional analysis

Metric and topological spaces. Convergence of sequences in metric spaces. Completeness and completion of metric spaces. Separability. The principle of compressive mappings. Compactness of sets in metric and topological spaces.

Normalized and topological linear spaces.

Linear spaces. Convex sets and convex functionals, the Banach-Hahn theorem. Separability of convex sets. Normalized spaces. Criteria for compactness of sets in the spaces C and Lp. Euclidean spaces. Topological linear spaces.

Linear functionals and linear operators. Continuous linear functionals. A general view of linear bounded functionals on basic function spaces. Conjugate space. Weak topology and weak convergence. Linear operators and their conjugates. The space of linear bounded operators. Spectrum and resolvent. Compact (completely continuous) operators. Fredholm's theorems.

Hilbert spaces and linear operators in them. Isomorphism of separable infinite-dimensional Hilbert spaces. Spectral theory of bounded operators in Hilbert spaces. Functional calculus for self-adjoint operators and the spectral theorem. Diagonalization of compact self-adjoint operators. Unlimited operators.

Differential calculus in linear spaces. Differentiation in linear spaces. Strong and weak differentials. Higher-order derivatives and differentials. Extremal problems for differentiable functionals. Newton's method.

Generalized functions. Regular and singular generalized functions. Differentiation, direct product, and convolution of generalized functions. Generalized slow-growth functions; their Fourier transform. Laplace transform of generalized functions (operational calculus). Structure of generalized functions with compact support.

4. Methodological recommendations for the preparation and passing of the final state exam

Recommended literature

a) Basic literature
Vladimirov V. S. Equations of Mathematical Physics. Moscow: Nauka, 1981.
Kolmogorov A. N., Fomin S. V. Elements of the theory of functions and functional analysis. Moscow: Nauka, 1976.
Lavrentiev M. A., Shabat B. V. Methods of the theory of functions of a complex variable. Moscow: Nauka, 1973.
Markushevich A. I. Theory of analytical functions. Vol. 1, 2. Moscow: Nauka, 1967-1968.
Natanson I. P. Theory of functions of a real variable. Moscow: Nauka, 1974.
Nikolsky S. M. Course of mathematical analysis. Vol. 2. Moscow: Nauka, 1983.
Privalov I. I. Introduction to the theory of functions of a complex variable. Moscow: Nauka, 1977.
Reid M., Simon B. Methods of modern mathematical physics. Vol. 1. Functional analysis. Moscow: Mir, 1976.
Rudin U. Fundamentals of mathematical analysis. Moscow: Mir, 1976.
Smirnov V. I. Course of higher mathematics. T. V. M.: Fizmatgiz, 1959.
Shabat B. V. Introduction to complex analysis. Part 1. Moscow: Nauka, 1969.
b) Additional literature
Dyachenko M. I., Ulyanov P. L. Measure and integral. M.: Factorial, 1998.
Evgrafov M. A. Analytical functions. M.: Nauka, 1991.
Zorich V. A. Mathematical Analysis, vol. 2. Moscow: Nauka, 1984.

Lyusternik L. A., Sobolev V. I. Elements of functional analysis. Moscow:
Nauka, 1965.
Rudin U. Functional analysis. Moscow: Mir, 1975.
Sadovnichy V. A. Theory of operators. Moscow: Higher School, 1999.
Hutson V., Pym J. Applications of functional analysis and Operator theory.
Moscow: Mir, 1983.

c) databases, information and reference systems and search engines

1. Higher Attestation Commission RF http://vak.ed.gov.ru

2. RSL Electronic Library http://www.rsl.ru/

3. RUDN Library http://lib.rudn.ru/

4. Science Direct http://www.sciencedirect.com Description: The resource contains a collection of scientific, technical full-text and bibliographic information. The multidisciplinary database includes scientific journals in the exact and technical fields of science.

5. EBSCO http://search.ebscohost.com, Academic Search Premier (a database of complex topics, containing information on the humanities and natural sciences).

6. Oxford University Press http://www3.oup.co.uk/jnls. Journals in the exact and technical sciences of Oxford University Press presented in the collection HSS

7. Sage Publications http://online.sagepub.com . The Sage publication database includes journals in various fields of knowledge: Sage_STM – more than 100 journals in the field of natural sciences, engineering.
8. Springer/Kluwer http://www.springerlink.com. Journals and books publishing houses
9. Springer/Kluwer cover various fields of knowledge and are divided into subject categories.
10. Tailor & Francis http://www.informaworld.com . The collection of journals includes more than 1000 titles in all fields of knowledge.
11. American Mathematical Society http://www.ams.org/ A resource of the American Mathematical Society.

12. European Mathematical Society http://www.euro-math-soc.eu/ Resource of the European Mathematical Society.

13. Portal to Mathematics Publications http://www.emis.de/projects/EULER/

14. Catalog of mathematical Internet resources http://www.mathtree.ru/

15. Zentralblatt MATH (zbMATH) https://zbmath.org

16. All-Russian mathematical portal mathnet.ru

17. Web of Science http://www.isiknowledge. com
18. Resources of the Institute of Scientific Information on Social Sciences of the Russian Academy of Sciences http://elibrary.ru.
19. University Information System RUSSIA. http://www.cir.ru/index.jsp.

20. GOST standards system for information, library and publishing http://www.ifap.ru/library/gost/sibid.htm.

21. RUDN Electronic Library http://www.rsl.ru/

d) periodicals

Algebra i analiz, Diskretnaya matematika, Zhurnal vychislitel'noj matematiki i matematicheskoj fiziki, Izvestiya Rossijskoj akademii nauk. Seriya matematicheskaya, Matematicheskie zametki Matematicheskij sbornik, Matematicheskoe modelirovanie, Teoreticheskaya i matematicheskaya fizika, Teoriya veroyatnostej i ee primeneniya, Uspekhi matematicheskih nauk, Funkcional'nyj analiz i ego prilozheniya, Trudy Matematicheskogo instituta im. V. A. Steklova, Sovremennye problemy matematiki, Vychislitel'nye metody i programmirovanie, Trudy seminara imeni I. G. Petrovskogo, Uchyonye zapiski Moskovskogo gosudarstvennogo universiteta Fundamental'naya i prikladnaya matematika, Review of Modern Physics, Review of Modern Physics, Annual Review of Astronomy and Astrophysics, Annual Review of Biochemistry, Chemical Reviews Nature Physics, Annual Review of Condensed Matter Physics, Annals of Mathematics, Journal of the American Mathematical Society, Acta Mathematica, Communications on Pure and Applied Mathematics Swarm and Evolutionary Computation Geometric and Functional Analysis Formal Aspects of Computing, Discrete Mathematics, Theory of Computing Systems Reports on Progress in Physics New Journal of Physics.

5. Estimated funds

The estimated funds are presented in the form of the estimated funds fund in the annex to this program.

6. Requirements for the scientific report on the main results of the scientific qualification work

6.1. A person who has successfully completed the full development of the main educational program for training scientific and pedagogical personnel in graduate school and has successfully passed all other types of final certification tests is allowed to defend a scientific report on the main results of scientific and qualification work before the SAC.

6.2. The defense of the scientific report on the main results of the scientific qualification work before the SAC is carried out in the form of an oral presentation of the scientific report on the main results of the SQW with subsequent oral responses to the comments of the reviewer and to the questions of the members of the State Attestation Commission in accordance with the Regulations on the state final certification of educational programs of higher education — programs for training scientific and pedagogical personnel in postgraduate studies, approved by the Rector's Order No. 41 of 20.01.2017.

6.3. As part of the defense of the scientific report on the main results of the SQW, the degree of development of the following competencies by students is checked

universal competencies:

- willingness to participate in the work of Russian and international research teams to solve scientific and scientific-educational tasks (UC-3);

general professional competencies
- knowledge of the methodology of theoretical and experimental research in the field of professional activity (GPC-1);

List of sample topics of research and qualification works

In accordance with paragraph 32. "Regulations on the state final certification of educational programs of higher education-programs for the training of scientific and pedagogical personnel in postgraduate studies", approved by the Rector's Order No. 41 of 20.01.2017, the approximate list of topics of the SQW is determined by the graduating department / educational department and approved by the dean of the faculty / director of the Institute/ director of the Academy. The student is informed of the approximate list of the NKR within 3 months from the moment of his admission to the postgraduate school.

Approximate list of topics of the SQW of students of the year of admission in 2020, who will go to the defense of the SQW in the 2024-2025 academic year.:

Singular integrals in Morrie-type spaces

Investigation of the absence of solutions to some partial differential inequalities using complex analysis methods

Embedding theorems and trace theorems for Nikolsky-Besov-Morrie spaces

The final name of the SQW can be approved by the presentation of the graduating department / educational department at the Academic Council of the faculty / institute/ Academy no later than 2 months before the submission of the scientific report on the main results of the SQW to the SAC.

The stages of the scientific qualification work, the conditions for the student's admission to the procedure for defending the scientific report on the main results of the SQW, the requirements for the structure, volume, content and design of the SQW and the scientific report on the main results of the SQW are determined by the "Regulations on the state final certification of educational programs of higher education - programs for the training of scientific and pedagogical personnel in postgraduate studies", approved by the Rector's Order of 20.01.2017. No. 41, "Regulations for the preparation and registration of scientific qualification work (dissertation) on the programs of training of highly qualified personnel at the Peoples 'Friendship University of Russia", approved by the Rector's Order No. 40 of 20.01.017, as well as GOST R 7.0.11-2011 " Dissertation and abstract of the dissertation. Structure and rules of registration".

The defense of the scientific report on the main results of the scientific qualification work is held at an open meeting of the state attestation commission with the participation of at least two-thirds of its members.

The presentation and discussion of the scientific report on the main results of the SQW as a state certification test is of the nature of a scientific discussion and is conducted in accordance with the following regulations:

* presentation of the student with a scientific report on the main results of the SQW (up to 15 minutes);

* the student's answers to the reviewer's comments and questions about the scientific report;

* making and announcing the decision of the state attestation commission on the results of the state attestation test in the form of a scientific report on the main results of the SQW.

The results of the defense of the scientific report on the main results of the NKR are determined by the ratings "excellent", "good", "satisfactory", "unsatisfactory". The decision of the state examination commission is made at a closed meeting of the commission and is announced on the day of submission of the report.

The interpretation of the evaluation criteria and the compliance of the scores with the assessments in the part of the defense of the scientific report on the main results of the scientific qualification work is presented in the annex to this program in the form of a evaluation fund.

The program is compiled in accordance with the requirements of the ES HE PFUR.

Director of direction
01.06.01 Mathematics and Mechanics
The Director of the S. M. Nikolsky
Mathematical Institute _____ Skubachevsky A. L.