

*Federal State Autonomous Educational Institution of Higher Education  
Peoples' Friendship University of Russia»*

*Faculty of Physical, Mathematical and Natural Sciences*

**WORKING PROGRAM OF THE DISCIPLINE**

**Name of the discipline**

**Numerical study of mathematical models**

**Recommended for the training area/specialty**

01.04.01 Mathematics

**Program focus (profile)**

"Functional Methods in Differential Equations and Interdisciplinary Research (eng.)»

### 1. Goals and objectives of the discipline:

to form an idea of numerical methods of research of mathematical models and areas of application of these methods, to improve the student's mathematical culture. The implementation of this goal includes a consistent presentation of theoretical material in lectures, in which all the main results are provided with strict proofs; working out methods for solving problems in practical classes; intermediate and final control reveal the degree of assimilation of the acquired skills.

### 2. Place of the discipline in the structure of OOP:

The discipline "Numerical methods for the study of mathematical models" refers to *the subjects chosen by the student of module 2* of the curriculum.

Table №1 shows the previous and subsequent disciplines aimed at the formation of the discipline's competencies in accordance with the matrix of competencies of the Higher Professional Education Department.

Table № 1

#### Previous and subsequent disciplines aimed at developing competencies

/	Code and name of the competence	Previous discipline	Subsequent disciplines (groups of disciplines)
General professional competencies			
	<b>OPK-1.</b> Able to formulate and solve actual and significant problems in mathematics	Modern problems of mathematics and applied mathematics	Introduction to low-dimensional topology, Operators in Function spaces, State Exam

### 3. Requirements for the results of mastering the discipline

As a result of studying the discipline, the student must:

#### Know:

Basic concepts of the theory of difference schemes, properties of difference schemes.

#### Be able to:

Replace boundary value problems with stable difference schemes of the required approximation order

#### Own:

techniques and methods of analysis of difference schemes and their application for numerical research: mathematical models.

### 4. Scope of the discipline and types of academic work

The total labor intensity of the discipline is 3 credits.

№	Type of academic work	Total hours	Modules			
			1	2	3	4
<b>1.</b>	<b>Classroom classes (ac. hours)</b>					48
	Including:					
1.1.	Lectures					16
1.2.	Other activities					32
	Including:					
1.2.1.	<i>Practical exercises (PE)</i>					
1.2.2.	<i>Seminars (S)</i>					32
1.2.3.	<i>Laboratory work (LW)</i>					

<i>Of these, in interactive form (IF):</i>						
<b>2.</b>	<b>Independent work of students (ak. hours)</b>					60
	Including:					
2.1.	Course project (work)					
2.2.	Calculation and graphic works					
2.3.	Summary					
2.4.	Preparation and passing of intermediate certification					36
	<i>Other types of independent work</i>					24
<b>3.</b>	<b>Total labor intensity (ac. hours)</b>					108
	<i>Total labor intensity (credits)</i>					3

### 5.1. Content of discipline sections

	Name of the discipline section	Section content (topics)
<b>1.</b>	<b>Difference schemes for boundary value problems</b>	Difference schemes for boundary value problems for second-order ODES with variable coefficients. Solution method. Approximation of boundary value problems for second-order ODES with variable coefficients by difference schemes. Stability of difference schemes and convergence of the approximate solution to the exact one
<b>2.</b>	<b>Discrete Fourier series</b>	Discrete Fourier series Discrete Fourier series. Approximate solution of the initial-boundary value problem for the heat conduction equation based on discrete Fourier series
<b>3.</b>	<b>Difference schemes for the heat conduction equation</b>	The simplest difference schemes for the heat conduction equation. Explicit schema. Conditional stability of an explicit scheme for the heat conduction equation.
<b>4.</b>	<b>Diagram with weights for the heat conduction equation</b>	Diagram with weights for the heat conduction equation. Approximation of the scheme with weights. High-order precision scheme. Conditional and unconditional convergence of difference schemes for the heat conduction equation.
<b>5.</b>	<b>Solution of the Dirichlet problem for the Laplace equation</b>	Solution of the Dirichlet problem for the Laplace equation as an establishment problem. Scheme of variable directions.
<b>6.</b>	<b>Solving problems for hyperbolic equations</b>	Difference schemes for solving problems for hyperbolic equations. Cross diagram

### 5.2. Discipline sections and types of classes

	Section name	Lectures	Practical exercises and laboratory work	SIW	Total

			PE / S	LR	from them in IF		
1.	Difference schemes for boundary value problems	2	4			10	16
2.	Discrete Fourier series	2	4			10	16
3.	Difference schemes for the heat conduction equation	3	6			10	19
4.	Diagram with weights for the heat conduction equation	3	6			10	19
5.	Solution of the Dirichlet problem for the Laplace equation	3	6			10	19
6.	Solving problems for hyperbolic equations	3	6			10	19
	<b>TOTAL</b>	16	32			60	108

## 6. A laboratory workshop –is not provided

## 7. Practical exercises (seminars)

	Name of the discipline section	Just an hour.
1.	Difference schemes for boundary value problems	4
2.	Discrete Fourier series	4
3.	Difference schemes for the heat conduction equation	6
4	Diagram with weights for the heat conduction equation	6
5	Solution of the Dirichlet problem for the Laplace equation	6
6	Solving problems for hyperbolic equations	6

## 8. Material and technical support of the discipline:

classroom for seminars, lecture hall, laptop - 1 pc., projector-1 pc., screen-1 pc., copier - 1 pc., printer-1 pc., scanner-1 pc.

## 9. Informational support of the discipline

a) software: not required

b) databases, information and reference systems and search engines: Yandex, Google, MathNet.

## 10. Educational, methodical and informational support of the discipline:

a) basic literature:

1. Kalitkin N. N. Chislennyye metody [Numerical methods]. Kalitkin, St. Petersburg: BHV, 2014, 592 p. .
2. Samarskii A. A., Gulin A.V. Chislennyye metody matematicheskoi fiziki [Numerical methods of mathematical physics].
3. Bakhvalov N. S., Lapin A.V., Chizhonkov E. V. Chislennyye metody v zadachakh i prakasheniyakh: Uchebnoe posobie [Numerical methods in problems and exercises: A textbook].

b) additional literature:

5. Bakhvalov, N. S. Chislennye metody [Numerical methods]./ Bakhvalov N. S., Zhidkov N. P., Kobelkov G. M.

Moscow: BINOM. Knowledge lab. 2004, 636 p.

### **11. Methodological recommendations for organizing the study of the discipline:**

The course is taught in the form of practical exercises. This course is based on mathematical analysis, ordinary differential equations, partial differential equations, and numerical methods.

Two test papers and one colloquium are held per semester. The colloquium requires knowledge of basic concepts, definitions, and statements of theorems without proof. The final control of knowledge implies a more in-depth knowledge and understanding of numerical methods for studying mathematical models

### **12. Fund of assessment funds for conducting intermediate certification of students in the discipline (module): attached.**

The program is designed in accordance with the following requirements: OS 3++ RUDN University.

#### **Developers:**

ProfessioniorMath Department

S. M. Nikolsky Institute  
position, department name



E.B. Laneev

#### **Head of the program:**

Professor, S.M.Nikolskii MI



Burenkov V.I.

#### **Director**

S.M.Nikolskii MI



Skubachevskii A.L.

**FEDERAL STATE AUTONOMOUS EDUCATIONAL INSTITUTION OF  
HIGHER EDUCATION  
PEOPLES' FRIENDSHIP UNIVERSITY OF RUSSIA  
(RUDN UNIVERSITY)  
Faculty of Physical, Mathematical and Natural Sciences**

**S. M. Nikolsky Mathematical Institute**

APPROVED

at the Institute's meeting

" \_\_\_ " \_\_\_\_\_ 20\_\_ city, protocol no.

\_\_\_\_\_  
Director of the Institute

\_\_\_\_\_ A. L. Skubachevsky  
(signature)

# **EVALUATION FUND**

**BY ACADEMIC DISCIPLINE**

Numerical research of  
mathematical models  
(name of the discipline)

01.04.01 «Mathematics»

master's degree  
Graduate qualification (degree)

# Passport of the evaluation fund for the discipline "Numerical research of mathematical models"

To the Management Board/Specialty: 01.04.01 " Mathematics»

Code of the controlled competence or part of it	Controlled section of the discipline	Controlled topic of the discipline	Name of the evaluation tool														Points of the topic	Points of the section		
			Current monitoring											Intermediate certification						
			Survey	Test work	Colloquium	Test work	Performing the LR	Implementation of CBL/CP	SIW (Performing a home test)	Summary	RGR execution	::	::	::	Exam	::			::	
OPK-1	Section 1: "Diagram with weights for the heat transfer equation the heat transfer equation "»	Topic 1: "The simplest difference schemes for the heat conduction equation. Explicit schema. Conditional stability of an explicit scheme for the heat conduction equation»"			2,5	5										12,5			20	47,5
		Topic 2 : " Scheme with weights for the heat conduction equation. Approximation of the scheme with weights. High-order precision scheme. Conditional and unconditional convergence of difference schemes for the heat conduction equation»	2,5		2,5	10										12,5			27,5	

	Section 2: "Solution of the Dirichlet problem for the Laplace equation"	Topic 1: "Solution of the Dirichlet problem for the Laplace equation as a problem for establishing. Variable direction diagram»"	2,5		5	5								12,5			25	52,5
		Topic 2: "Difference schemes for solving problems for hyperbolic equations. Cross diagram»"				10			5						12.5			27.5
	<b>TOTAL:</b>	5		10	30			5						50			100	100



## EXAM TICKETS

### Discipline *Comprehensive analysis*

#### EXAM TICKET # 1

1. Difference schemes for boundary value problems for second-order ODES with variable coefficients. Solution method.
2. Conditional stability of an explicit scheme for the heat conduction equation.

#### EXAM TICKET # 2

1. Approximation of boundary value problems for second-order ODES with variable coefficients by difference schemes.
2. Diagram with weights for the heat conduction equation

#### EXAM TICKET # 3

1. Stability of difference schemes and convergence of the approximate solution to the exact one.
2. Approximation of the scheme with weights.

#### EXAM TICKET # 4

1. Discrete Fourier series.
2. High-order precision scheme.

#### EXAM TICKET # 5

1. Approximate solution of the initial-boundary value problem for the heat conduction equation based on discrete Fourier series
2. Conditional and unconditional convergence of difference schemes for the heat conduction equation.

#### EXAM TICKET # 6

1. The simplest difference schemes for the heat conduction equation. Explicit schema.
2. Solution of the Dirichlet problem for the Laplace equation as an establishment problem.

#### EXAM TICKET # 7

1. Difference schemes for boundary value problems for second-order ODES with variable coefficients. Solution method.
2. Solving problems for hyperbolic equations.

#### EXAM TICKET # 8

1. Approximation of boundary value problems for second-order ODES with variable coefficients by difference schemes.
2. The simplest difference schemes for the heat conduction equation. Explicit schema

**Each student gets one ticket from this list. The answer to each question is rated from 0 to 25 points, depending on the completeness and correctness of the answers.**

## CONTROL WORK OPTIONS

### Test work # 1

Option 1.

Write a difference scheme for the first boundary value problem for a second-order ODE:

$$(x u')' - 2u = -2, \quad u(0) = 1, \quad u(1) = 2$$

Option 2.

Write a difference scheme for the first boundary value problem for a second-order ODE:

$$(xx u')' - u = 4x - xx, \quad u(1) = 1, \quad u(2) = 4$$

### Test work # 2

Option 1.

Write a difference scheme with weights for the heat conduction equation at  $\sigma = 1/2$ .

Option 2.

Write a difference scheme with weights for the heat conduction equation at  $\sigma = 1$ .

**Each task of the test papers is rated from 0 to 3 points, depending on the completeness and correctness of the solution.**

### QUESTIONS FOR THE COLLOQUIUM

1. Difference approximations for first-order derivatives. The order of approximation.
2. Difference approximations for second-order derivatives. The order of approximation.
3. Order of approximation of the difference scheme.
4. Stability of the difference scheme
5. Conditional stability of the difference scheme.
6. Orthogonality of a system of functions of a discrete Fourier series.
7. Explicit and implicit scheme for the heat conduction equation.
8. Scheme with weights
9. Scheme of increased order of accuracy.
10. Conditional and unconditional convergence of difference schemes for the heat conduction equation.

**Each student gets one question from this list. The answer to each question is rated from 0 to 10 points, depending on the completeness and correctness of the answers.**

### SET OF SURVEY QUESTIONS

- 1) What is the order of approximation of a difference scheme?
- 2) What is the stability of a difference scheme?
- 3) What does the expression "Convergence follows from approximation and stability"?
- 4) What is conditional stability of a difference scheme?
- 5) Hence the completeness of the system of functions of a discrete Fourier series.
- 6) Why do I need a scheme with weights?
- 7) What is an explicit difference scheme?
- 8) What is not explicit difference scheme?