Federal state autonomous educational institution of higher education
«Peoples' Friendship University of Russia »

Faculty of science
Recommended by MSSN

PROGRAM OF DISCIPLINE

Additional chapters of functionsl analisys

Recommended for

01.06.01 «Mathematics and mechanics»

Profile

«Real, complex and functional analysis»

1. Aims and problems.

Aim of the discipline “Additional chapters of functionsl analisys ” is to teach students in some modern methods of the function theory and mathematical methods that are applied for study the modern problem of this theory

2. Place of discipline in educational structure.

Table 1 contains the preceding and following disciplines for the forming of needed competencies.

Table 1.

The preceding and following disciplines for the forming of needed competencies.

№	Names of competitions	Preceding disciplines	following disciplines
Professional competitions			
	PC-1. The knowledge in main sections of real, complex and functional analysis, including measure theory, theory of Lebesgue integral, Fourier series, Fourier transforms theory of analytic functions, theory of metric spaces, Banach and Gilbert spaces, theory of bounded and compact operators in Banach spaces, theory of self-adjoined operators, spectral theory of operators. PC-3: the ability to formulate the aim of research and the methods of realization, generalization of the results and the ability to make the corresponding conclusions. To understand the practical aspects of theoretical results.	-	General theory of functional-differential equations. Nonlinear partial equations. Variational analysis in differential equations. Mathematical methods in economics
Universal Competitions			
	UC-1. The ability to critical analysis of modern scientific results, generation on new ideas for the solutions of researches, and practical problems, in particular in inter-disciplinal domains. UC-2. The ability to realize complex researches, in particular, in inter-	-,	General theory of functional-differential equations. Methods of optimal control. Mathematical methods in economics.

	disciplinal domains on the base of systematical scientific views. UC-3. Ability to participate in Russian and international teams for the realization of scientific and educational problems. UC-5. The ability to plane and realize problems of own professional and personal development.		

3. The requirements to the results:
The studying is concentrated on formation of the following competitions: UC-1, UC-2, UC-3, UC-5, PC-1, and PC-3. As the result of studying, the student has

to know: the main types of functional spaces,

– the criteria of boundedness for the main operators of harmonic analysis in different function spaces;
– the grounds of the theory of optimal embeddings;
-the main result on traces and extensions for the main function spaces.
to be able:
- to formulate and to study the problems of approximation theory;
- to formulate the problems of optimal embeddings;
- to formulate and to study the problems on estimates of the norms of integral operators in functional paces;
to choose the methods of rewearches.

To posses: the modern mathematical methods for obtain the solutions of the problems in the theory of function spaces.

4. The volume of discipline and branches of studying
The complete volume of the course “Additional chapters of functional analysis” contains 4 units.

Forms of studying	Volume in hours	Semesters			
Lessons (general)		1	2		
In particular:	-	-	-	-	-
Lectures	40	20	20		
Practical lessons (PL)	40	20	20		
Seminars (S)					
Laboratories (L)					
Independent studies (general)	64	32	32		
The whole volume: hours	144	72	72		
units.		2	2		

5. Contents.

5.1. Contents of sections.

1 Semester

№	Title of section	Contents of section
1.	Definition and general properties of the space L_p, $0 < p \le \infty$. Properties of norm and quasi-norm	Main concepts of Lebesgue measure and integral ıeory. Chebyshev inequality and its corollaries. 'eorems about limiting passage in Lebesgue integral: Levy, Lebesgue and Fatou. Definition and eneral properties of the space L_p, $0 < p \le \infty$. Convergence and completeness.
2.	Holder inequality.	The proof of Holder inequality. The sharpness of Holder inequality. Absence of linear continuous functionals in the space L_p, $0 < p < 1$. Applications of Holder inequality: embeddings of the spaces on the sets with finite measures, multiplicative inequality and discrete Holder inequality.
3.	Minkowski inequality	Minkowski inequality for $p \ge 1$, property of the norm. Minkowski inequality for $0 < p < 1$, property of quasi-norm. The sharpness of constants in Minkowski inequality. The generalized Minkowski inequality for sums in the case $p \ge 1$.The generalized Minkowski inequality for sums in the case $p \ge 1$. The generalized Minkowski inequality for sums in the case $0 < p < 1$. The generalized Minkowski inequality for integrals. The generalized Minkowski inequality for convolutions. Young inequality.
4.	Hardy inequality	Conclude of Hardy inequality. Sharpness of conditions for its validity. Sharpness of the constant in Hardy inequality (calculation of the norm of Hardy operator).
5.	The basic concepts of interpolation theory. Classical Riesz-Torine theorem and its application in Fourier transform theory.	Fourier transform for functions from L_p. Plancherel theorem and Parseval equality in L_2. Hausdorf -Young and Paley-type theorems and their establishments with the methods of interpolation.
6	Definition and main properties of Fourier multipliers	Space of Fourier multipliers in L_p. The sufficient condition of Fourier multiplier in L_p - integrability of its Fourier transform. Formulation and the proof of the main Theorem about Fourier multipliers.
7.	Theorems about Fourier multipliers and their applications.	Marcinkievich, Lizorkin, and Mikhlin-Hormander theorems on Fourier multipliers. M. Riesz theorem on boundedness of Dirichlet sums. Characteristic functions of the band, rectangular and convex polyhedron as Fourier multipliers. The result of Charles Fefferman on the problem of jf Fourier multipliers.

8.	Subspaces of functions with restricted support of spectrum in Lebesgue space	Integral representation for functions with restricted support of spectrum. The main integral inequalities for functions with restricted support of spectrum. Estimates for derivatives, inequality for different metrics. Traces of functions and inequality of different dimensions for functions with restricted support of spectrum
9	Best approximations for functions from Lebesgue spaces by the functions with restricted support of spectrum.	Concept of the best approximation? Its general properties. Realization of best approximations in the space L_2. Algorithm for construction of "order-sharp" approximation, its connection with the theory of Fourier multipliers. .
10.	Approximation theorems for integrable functions by infinitely differentiable functions.	Sobolev averaging of functions, its relation with the generalized differentiation. The construction of approximation by compactly supported infinitely differentiable functions. Sobolev space, its completeness.

2 Semester

№	Title of Section	Contents of section
1.	Theory of Sobolev spaces for domains with cone condition.	Sobolev's integral representation and its corollaries. Estimates for intermediate derivatives for functions from Sobolev space. Application of Young and Hardy-Littlewood-Sobolev inequalities. Embeddings with different metrics for Sobolev spaces.
2.	Properties of moduli of continuity as nonlinear fractional characteristics of smoothness.	Properties of differences and moduli of continuity as nonlinear fractional characteristics of smoothness. Monotonicity properties and estimates for moduli of continuity. Marshoud theorem on the relations between moduli of continuity with different order.
3.	General properties of spaces with fractional smoothness. Classical Nikolskii-Besov spaces. Spaces with generalized smoothness.	Definitions of Nikolskii-Besov spaces, their general properties. Equivalent norms in Nikolskii-Besov spaces in terms of moduli of continuity in integral and discrete forms. Completeness of Nikolskii-Besov spaces. Generalizations of Nikolskii-Besov spaces – space with generalized smoothness.
4.	Embeddings for Nikolskii-Besov spaces without change of metrics.	The main lemma on estimates of discrete sums. Embedding theorems with respect to smoothness index and to second index. Scale of Nikolskii-Besov spaces. Estimates of modulus of continuity for locally singular function. Criteria of its belonging to Sobolev, Nikolskii-Besov spaces, and spaces with generalized smoothness.
5.	Characterization of generalized derivatives in terms of differences.	Averaging operators, properties of their kernels. Integral representations of differences through derivatives. Theorems on properties of averaging operators. Criteria for belonging of generalized

		directional derivatives to L_p in terms of differences.
6.	Characterization of Sobolev space in terms of differences. Relation between Sobolev and Nikolskii-Besov spaces.	Equivalent norms and characterization of Sobolev spaces in terms of differences. Mutual embeddings between Sobolev and Nikolskii-Besov spaces.
7.	Sharp description for space of traces for functions from Sobolev space on subspaces with smaller dimension.	Trace theorem for functions from Sobolev space. Theorems on extension. Coincidence of the trace-space with Besov space on the boundary. The posing of boundary problems in Sobolev classes.
8.	Inclusion of derivatives in norms of Nikolskii-Besov spaces and their generalizations.	Equivalent norms in generalized Nikolskii-Besov spaces by using generalized derivatives and moduli of continuity.
9.	Embedding with different metrics for generalized Nikolskii-Besov spaces.	Equivalent norms in generalized Nikolskii-Besov spaces in terms of extensions in series by functions with compact support of spectrum. Formulation and proof of embedding theorem with different metrics. The sharpness of embedding theorem.
10.	Sharp description for space of traces for functions from Nikolskii-Besov space on subspaces with smaller dimension.	The notion of the trace on subspaces with smaller dimension. Theorem about exact description for space of traces for Nikolskii-Besov spaces. Limiting case for this theorem. The non-existence of linear extension operators in limiting case.

***5.2.* Sections and forms of lessons**

1 Semester

№ п/п	Title of Section	Lectures.	Seminars	Independent work	All
1.	Definition and general properties of the space L_p, $0 < p \leq \infty$. Properties of norm and quasi-norm	4	4	6	14
2.	Holder inequality.	2	2	4	8
3.	Minkowski inequality	2	2	4	8
4.	Hardy inequality	2	2	4	8
5.	Basic concepts of interpolation theory. Riesz- Thorin theorem and its application in the theory of Fourier transform	2	2	3	7
6.	Definitions and main properties of Fourier multipliers	2	2	3	7
7.	Theorems on Fourier multipliers and their applications	2	2	4	8

8.	Subspaces of functions with restricted support of spectrum in Lebesgue space.	2	2	2	6
9.	Best approximations for functions from Lebesgue spaces by the functions with restricted support of spectrum. .	1	1	1	3
10.	Approximation theorems for integrable functions by infinitely differentiable functions.	1	1	1	3
	In all:	20	20	32	72

2 Semester

1.	Theory of Sobolev spaces for domains with cone condition.	4	4			6	14
2.	Properties of moduli of continuity as nonlinear fractional characteristics of smoothness.	2	2			4	8
3.	General properties of spaces with fractional smoothness. Classical Nikolskii-Besov spaces. Spaces with generalized smoothness.	2	2			4	8
4.	Embeddings for Nikolskii-Besov spaces without change of metrics.	2	2			4	8
5.	Characterization of generalized derivatives in terms of differences.	2	2			3	7
6.	Characterization of Sobolev space in terms of differences. Relation between Sobolev and Nikolskii-Besov spaces.	2	2			3	7
7.	Sharp description for space of traces for functions from Sobolev space on subspaces with smaller dimension	2	2			4	8
8.	Inclusion of derivatives in norms of Nikolskii-Besov spaces and their generalizations.	2	2			2	6
9.	Embedding with different metrics for generalized Nikolskii-Besov spaces.	1	1			1	3
10.	Sharp description for space of traces for functions from Nikolskii-Besov space on subspaces with smaller dimension.	1	1			1	3
	In all:	20	20			32	72

6. Laboratories are not planed

7. Seminars.

1 Semester

№	№ of Section	Titles of seminars	Volume (hours)

1.	**1**	Definition and general properties of the space L_p, $0 < p \leq \infty$. Properties of norm and quasi-norm	2
2.	**2**	Holder inequality.	2
3.	**3**	Minkowski inequality	2
4.	**4**	Hardy inequality	2
5.	**5**	Basic concepts of interpolation theory. Riesz- Thorin theorem and its application in the theory of Fourier transform	2
6.	**6**	Definitions and main properties of Fourier multipliers	2
7.	**7**	Theorems on Fourier multipliers and their applications	2
8.	**8**	Subspaces of functions with restricted support of spectrum in Lebesgue space	2
9.	**9**	Best approximations for functions from Lebesgue spaces by the functions with restricted support of spectrum.	2
10.	**10**	Approximation theorems for integrable functions by infinitely differentiable functions.	2

2 Semester

№	№ of Section	Titles of seminars	Volume (hours)
11.	**1**	Theory of Sobolev spaces for domains with cone condition.	2
12.	**2**	Properties of moduli of continuity as nonlinear fractional characteristics of smoothness. .	2
13.	**3**	General properties of spaces with fractional smoothness. Classical Nikolskii-Besov spaces. Spaces with generalized smoothness. .	2
14.	**4**	Embeddings for Nikolskii-Besov spaces without change of metrics	2
15.	**5**	Characterization of generalized derivatives in terms of differences.	2
16.	**6**	Characterization of Sobolev space in terms of differences. Relation between Sobolev and Nikolskii-Besov spaces.	2
17.	**7**	Sharp description for space of traces for functions from Sobolev space on subspaces with smaller dimension	2
18.	**8**	Inclusion of derivatives in norms of Nikolskii-Besov spaces and their generalizations.	2
19.	**9**	Embedding with different metrics for generalized Nikolskii-Besov spaces.	2
20.	**10**	Sharp description for space of traces for functions from Nikolskii-Besov space on subspaces with smaller dimension.	2

8. Technical equipment:

Rooms 495a, 398, 509 in RUDN study building, Ordzhonikidze str., 3; group rooms in RUDN study building, Ordzhonikidze str., 3 (2, 3, and 4th floors), computer classes, laboratories (rooms 510 and 424).

9. Informational equipment:

Only licensed software installed at RUDN is used:
- Microsoft Office program package;
- Multimedia equipment and personal computers;
- Full-text databases and resources accessible from RUDN net;
- RFBR electronic library http://www.rfbr.ru/rffi/ru/library

10. Literature :

a) main literature:

1. S. M. Nikolsky. Approximation of functions of many variables and embedding theorems. Moscow: Nauka, 1977.
2. O. V. Besov, V. P. Ilyin, S. M. Nikolsky. Integral representations of functions and embedding theorems. Moscow: Nauka, 1996.
3. L. Hermander. Estimates for shift-invariant operators. Moscow: IL, 1962.
4. I. Stein. Singular integrals and differential properties of functions. Moscow: Mir, 1973
5. V. I. Burenkov. Functional spaces. Sobolev spaces. Moscow: RUDN, 1991.
6. T. A. Leontieva, V. S. Parfenov, V. S. Serov. Problems in the theory of functions of a real variable. Moscow: MSU, 1997.

б) additional literaturea:

7. V. G. Mazya. Sobolev spaces. LSU, 1985.
8. G. Tribel. The theory of interpolation. Functional spaces. Differential operators. Moscow: Mir, 1980.
9. S. G. Krein, Yu. I. Petunin, E. M. Semenov. Interpolation of linear operators. M. Nauka, 1978.
10. C. Bennett, R. Sharpley " Interpolation of operators”, Pure and Ap-plied Mathematics, V. 129 ,Academic Press, 1988.

11. Methodical recommendations for students

At seminars, key ideas of basic text sources of the course are presented. Namely, a student chooses a key idea of the text under discussion, formulates in theses (1–1.5 pages) its understanding and assessment, then presents and defends this at the seminar. The theses are distributed among the participants of the seminar in advance.

An essay should be written on a topic approved by the teacher. Its volume should not exceed 15 thousand symbols including spaces. An essay may consist in translating a paper of a foreign author with its extensive critical assessment and analysis. The author and the text must be approved by the teacher.

An exam takes place in the end of the semester in the form of an essay on one of the topics suggested by the teacher. After an interview a final note is given. The note is determined by intermediary assessment with notes «excellent», «good», «satisfactory», «unsatisfactory» and in the ECTS system (A, B, C, E). The notes are based on the RUDN score rating system.

12. The fund of evaluation funds for conducting intermediate certification of students in the discipline (module)

Materials for assessing the level of development of educational material of the discipline "Additional chapters of functional analysis" (evaluation materials), which include a list of competencies indicating the stages of their formation, a description of indicators and criteria for evaluating competencies at various stages of their formation, a description of assessment scales, standard control tasks or other materials necessary for evaluating knowledge, skills, skills and (or) experience of activities that characterize the stages of competence formation in the process of mastering the educational program, methodological materials defining the procedures for evaluating knowledge, skills, skills and (or) experience activities that characterize the stages of competence formation are fully developed and are available to students on the discipline page in the TUIS PFUR.

The program is compiled in accordance with the requirements of the ES HE PFUR.

Author:
Professor of the S. M. Nikolsky Mathematical Institute ____________ Goldman M.L.

The Director of the S. M. Nikolsky Mathematical Institute ____________ Skubachevsky A. L.