Federal state autonomous educational institution of higher education
«Peoples' Friendship University of Russia»

<u>*Faculty of science*</u>

Recommended by MSSN
"Mathematics"

Operative program of the subject

<u>Elements of perturbation theory</u>

Recommended for academic field

<u>01.04.01 «Mathematics»</u>

Profile of the program:
"Functional methods in differential equations and interdisciplinary research (Engl.)»

1. Aims and purposes of the subject: to master the main concepts and some mathematical methods of analysis of change of a compact operator's spectrum under small perturbations.

2. Position of the subject in the structure of the higher educational program:

The course «Elements of perturbation theory» is an elective course of part 1 in the study plan.

Table № 1 names preceding and consequent subjects aimed at forming competences of the subject according to the competence matrix.

Table № 1
Preceding and consequent subjects aimed at forming competences

N	Code and name of the competence	Preceding subjects	Consequent subjects
General professional competences			
	GPC-1 ability to formulate and solve actual and significant problems of mathematics	Functional analysis. Function spaces	Operators in function spaces

3. Requirements to the result of studies:
Studying the course is aimed at forming following competences: GPC-1

As a result of studying the course the student must:
Know: The basic concepts of mathematical analysis in Banach spaces, concepts of adjoint space and adjoint operator, spectral representation of an operator, Jordan form of an operator, algebraic and geometrical multiplicity of an eigenvalue, behavior of the rang of a proper projector under continuous dependence on a parameter.
Be able to: Formulate and prove theorems, construct Jordan basis, find the first term of the asymptotics of eigenvalues under a perturbation of a compact operator, construct the Schmidt operator for model examples.
Master: The skills of practical use of the mathematical methods under study.

4. Size of the course and types of studies
The course « Elements of perturbation theory» gives 4 credits.

Types of studies	Hours	Modules
		5
Class studies	54	54
Including:		
Lectures	27	27
Practical studies (PS)	27	27
Seminars (S)		
Laboratory work (LW)		
Home studies	63	63
Examination	27	27
Total, hours	144	144
Credits	4	4

5. Content of the subject

5.1. Content of the sections

№	Name of section	Content of section
1.	Analysis in operator space	Basis. Convergence and norms. Vector series. Adjoint space. Linear operator. Projector. Operator norm. Operator series. Operator-valued functions, their integration and differentiation.
2.	Spectral representation of a finite dimensional operator	Resolvent. Laurent series of a resolvent. Continuous dependence of eigenvalues of a finite dimensional operator on a small perturbation. Lipschitz behavior of eigenvalues under small commuting perturbation. Singularities of a resolvent. Spectral projector. Reduced resolvent. Canonical form of an operator.
3.	Method of Newton's diagrams for analysis of bifurcations of solutions to algebraic equations	Classical implicit function theorems. Asymptotical sequences and series. Newton's diagram. Bifurcation equation. Cases of simple and multiple roots. Definition of the principal term of the expansion. Specifying the asymptotics. Justification of convergence.
4	Bifurcation of eigenvalues	Jordan chains and sets of Fredholm operators. Condition of completeness of an A-Jordan set. Bifurcation of eigenvalues of Fredholm operators. Bifurcation equation. Method of indefinite coefficients.

5.2. Sections and types of studies

№	Section	Lect.	Practice			Home	Total
			Sem.	Exam.	In IF		
1.	Analysis in operator space	9	9	9		21	48
2.	Spectral representation of a finite dimensional operator	6	6	6		14	32
3.	Method of Newton's diagrams for analysis of bifurcations of solutions to algebraic equations	6	6	6		14	32
4.	Bifurcation of eigenvalues	6	6	6		14	32

6. Laboratory work – _none planned._

7. Practical classes (seminars)

№	№ of section	Themes of practical classes (seminars)	Hours
1.	**1**	Analysis in a normed space.	9
2	**2**	Operator space.	2
3	**2**	Reducing an operator to Jordan form.	4
4	3	Finding the first term of the asymptotics.	4
5	3	Specifying the asymptotics.	2
6	4	Constructing the Schmidt operator.	2
7	4	Bifurcation of eigenvalues.	4

8. Technical equipment:
Classrooms for lectures and seminars.

9. Informational equipment:
a) software – none required
b) databases, information and search systems
1. http://lib.rudn.ru/ - RUDN scientific library
2. http://techlibrary.ru/ - Technical library

10. Textbooks and methodical literature:
a) main literature
1. Ovchinnikov A.V., Kolybasova V.V., Krutitskaya N.Ch. Jordan form of operator matrix. 2009. MSU Publishers.
2. Kashchenko I.S. Asymptotic expansion for solutions of equations. YaSU Publishers. 2011.

b) additional literature
1. Kato T. Perturbation theory for linear operators. M.: Mir, 1972.
2. Vainberg M.M., Trenogin V.A. Bifurcation theory for solutions of nonlinear equations. Nauka, 1969.
3. Giacaglia G.E.O. Methods of perturbation theory for nonlinear systems. Nauka, 1979.

11. Methodical recommendations for students of the subject
Individual home assignments are given twice in the module. They should be completed in 4-5 weeks.
In the course of homework the following competences are formed:
- ability to abstract thinking, analysis, synthesis (GC-1);
- readiness to self-development, self-realization, using one's creative potential (GC-3);
- ability to intensive scientific research (PC-1).

Students who get less than half of maximal possible score for home assignments are required to attend consultations.

12. Repertory of estimates for intermediary attestation of the students in the subject

The subject is studied according to a credit-module system and is completed by an exam.
The forms of study are class work and home work of the students.
In the course of studied the following kinds of control are used: *current, boundary, intermediary.*

Current control consists in home tasks.
The students are obliged to assign all tasks at times determined by the teacher.

Boundary control takes place during the classes. The result of the boundary control is expressed in the number of the points collected by the student.

Intermediary control has the form of a written examination.
The knowledge of the students is estimated according to a 100-point system including the following parameters:
- Individual home assignments;
- Examination.

Structure and contents of the examination
Students with any number of points collected in the term are allowed to take examination.
The examination takes place in the written form. The assignment consists of 4 questions (3 theoretical ones and 1 problem).
The duration of the examination is 60 minutes.

Form of checking. The teacher checks the works within 2-3 hours and gives resulting points (the sum of points for home assignments and examination).

A student has the right to retake the examination if he gets less than 51 resulting points. In this case the points collected at the first examination are not taken into account, only those for the home assignments are. The examination is retaken in the periods from 07.02 till 28.02 in accordance with the dean's office.

General competences (GC):
 - ability to abstract thinking, analysis, synthesis (GC-1);
 - readiness to self-development, self-realization, using one's creative potential (GC-3)
 and *professional competences (PC):*
- ability to intensive scientific research (PC-1)
 are formed in the whole course of the studies.

Points are given according to PRS.

Summary evaluation table of the subject

Section	Topic	Competences	Forms of control of progress in the program												Points of the topic
			Test	Colloquium	Essay	Lab work	Homework	Calculation project	Term work	Term project	Classwork	Working int. class	Examination	Other froms of control	

Analysis in operator space	Topic 1: Analysis in normed spaces	GPC-1						5						5		10	25
	Topic 2: Operator space							10						5		15	
Spectral representation of a finite dimensional operator	Topic 3: Singularities of the resolvent	GPC-1						5						5		10	25
	Topic 4: Canonical form of an operator							10						5		15	
Method of Newton's diagrams for analysis of bifurcations of solutions to algebraic equations	Topic 5: Finding the first term of the asymptotics	GPC-1						5						5		10	25
	Topic 6: Specifying the asymptotics							10						5		15	
Bifurcation of eigenvalues	Topic 7: Constructing the Schmidt operator	GPC-1						10						5		15	25
	Topic 8: Bifurcation of eigenvalues							5						5		10	
	Total							60						40		100	100

Correspondence of note systems (notes for final academic progress used before, ECTS notes and point-rating system (PRS) of notes for current progress) – according to the Order of the Rector №996 dated 27.12.2006:

PRS points	Traditional notes in RF	Points for conversion of notes	Notes	ECTS notes
86 – 100	5	95 - 100	5+	A
		86 - 94	5	B
69 – 85	4	69 - 85	4	C
51 – 68	3	61 - 68	3+	D
		51 - 60	3	E
0 – 50	2	31 - 50	2+	FX
		0 - 30	2	F
51 – 100	Passed		Passed	Passed

Description of ECTS notes:

A («Excellent») – the theoretical content of the course is mastered completely, without gaps, the necessary practical skills of work with the learned stuff are formed, all assignments from the study program are completed, the quality of their completion is estimated by a number of points close to the maximum.

B («Very good») – the theoretical content of the course is mastered completely, without gaps, the necessary practical skills of work with the learned stuff are mostly formed, all assignments from the study program are completed, the quality of completion of their most part is estimated by a number of points close to the maximum.

C («Good») – the theoretical content of the course is mastered completely, without gaps, some practical skills of work with the learned stuff are not sufficiently formed, all assignments from the study program are completed, the quality of completion of none of them is estimated by a minimal number of points, some assignments are completed with mistakes.

D («Satisfactory») - the theoretical content of the course is mastered partially, but the gaps are not essential, the necessary practical skills of work with the learned stuff are mostly formed, all assignments from the study program are completed, some completed assignments may contain mistakes.

E («Mediocre») - the theoretical content of the course is mastered partially, some practical skills of work with the learned stuff are not formed, many assignments from the study program are not completed or the quality of completion of some of them is estimated by a number of points close to minimum.

FX («Conditionally unsatisfactory») - the theoretical content of the course is mastered partially, the necessary practical skills of work with the learned stuff are not formed, most assignments from the study program are not completed or the quality of completion of some of them is estimated by a number of points close to minimum; the quality of completion of assignments can be improved in the course of additional home work with the stuff of the course.

F («Definitely unsatisfactory») - the theoretical content of the course is not mastered, the necessary practical skills of work are not formed, all completed assignments contain gross mistakes, additional home work with the stuff of the course will not result in any significant improvement of the quality of completion of assignments.

Passport of repertory of evaluation instruments

Direction/Occupation: 01. 04. 01   Mathematics

Subject: Elements of perturbation theory

Code of controlled competence or its part	Controlled section of the subject	Controlled topic of the subject	FOS (forms of control of level of progress in the subject)	
			Class work	

			Poll		Colloquium		Lab work		Current homework	Essay	Project		Homework №1	Homework №2	Examination	…	.		
GPC-1	Analysis in operator space	Topic 1: Analysis in normed spaces											5	5				10	
		Topic 2: Operator space											10	5				15	25
GPC -1	Spectral representation of a finite dimensional operator	Topic 3: Singularities of the resolvent											5	5				10	25
		Topic 4: Canonical form of an operator											10	5				15	
GPC -1	Method of Newton's diagrams for analysis of bifurcations of solutions to	Topic 5: Finding the first term of the asymptotics										5	5				10	25
		Topic 6: Specifying the asymptotics											10	5				15	

	algebraic equations																								
GPC -1	Bifurcation of eigenvalues	Topic 7: Constructing the Schmidt operator															10	5			15	25
		Topic 8: Bifurcation of eigenvalues															5	5			10	
		TOTAL:													30	30	40			100	100

Subject «Elements of perturbation theory »

Examination paper № 1.
1. Adjoint operator.

2. Laurent series for a resolvent.

3. Find two first terms of asymptotic approximation for each solution of the equation
   ε^2*x^3 −εx + ε^2 = 0.

4. Schmidt operator.

_____

Examination paper № 2.
1. Nilpotent operator.
2. Take the matrix of the linear operator to the Jordan form

$$\begin{pmatrix} 4 & 1 & -1 \\ -2 & 4 & 5 \\ 1 & 0 & 1 \end{pmatrix}.$$

3. Asymptotic series.
4. A-Jordan chain of generalized eigenvectors.

In the framework of the examination the formation of all the competences of the subject can be checked (depending on the question).

The set of examination papers is supplied with estimation criteria in the subject elaborated by the teacher and approved at a meeting of the department.

Estimation criteria for replies to examination questions:
For a reply to each examination question, from 0 to 10 points are given:

Estimation criteria	Points		
	The reply does not meet the criterion	**The reply meets the criterion partially**	**The reply meets the criterion completely**
The reply is correct	0	1	2

The student replies without hints of the teacher	0	0,5	1
The student practically does not use the prepared draft	0	0,5	1
The reply shows that the student surely masters the terminology and methodology of the subject	0	1	2
The reply has a clear logical structure	0	1	2
The reply shows understanding of connections between the subject of the question and other sections of the course and/or with other courses	0	1	2

The program is prepared according to the requirements of OS 3++ RUDN.

Author:

Professor, S.M.Nikolskii MI   Galakhov E.I.

Head of the program:

Professor, S.M.Nikolskii MI   Faminskii A.V.

Director

S.M.Nikolskii MI   Skubachevskii A.L.