Federal State Autonomous Educational Institution of Higher Education
PEOPLES' FRIENDSHIP UNIVERSITY OF RUSSIA
RUDN University

Faculty of Science, S.M. Nikol’skii Mathematical Institute
educational division (faculty/institute/academy) as higher education programme developer

COURSE SYLLABUS

Generic functions
course title

Recommended by the Didactic Council for the Education Field of:

01.03.01 Mathematics
field of studies / speciality code and title

The course instruction is implemented within the professional education programme of higher education:

Mathematics
higher education programme profile/specialisation title

1. COURSE GOAL(s)

Possible wording

The course "Generic functions" aims to educate students in Generic functions and its applications to a wide range of different areas of modern analysis, topology and partial differential equations. This course deepens the mathematical culture of students and helps them master the fundamental mathematical concepts that form the basis of modern functional analysis, operator theory, harmonic analysis and related areas of real analysis. In acquainting students with the theoretical background, methods and concepts of Generic functions the course provides an introduction to the advanced study of more specialized topics in functional analysis, real analysis and partial differential equations.

2. REQUIREMENTS FOR LEARNING OUTCOMES

Possible wording

Mastering the discipline "Generic functions" expects students to acquire the following competencies:

Table 2.1. List of competences that students acquire through the course study

Competence code	**Competence descriptor**	**Competence formation indicators** (within this course)
GPC-1	Ability to apply the fundamental knowledge gained in the field of mathematical and (or) natural sciences and use it in professional activities	GPC-1.1. Uses existing and develops new methods of solving problems in mathematics GPC-1.2. Uses modern equipment, software and professional databases to solve mathematical problems
PC-1	Ability to determine Generic forms and rules of a separate subject area	PC-1.1. Plans individual stages of research in the presence of a Generic research plan PC-1.2. Prepares elements of documentation, draft plans and programs for individual stages of research PC-1.3. Selects research methods to solve the assigned research tasks

3. COURSE IN HIGHER EDUCATION PROGRAMME STRUCTURE

The course refers to the core/variable/elective* component of (B1) block of the higher educational programme curriculum.

* - Underline whatever applicable.

Within the higher education programme students also master other (modules) and / or internships that contribute to the achievement of the expected learning outcomes as results of the course study.

Table 3.1. The list of the higher education programme components/disciplines that contribute to the achievement of the expected learning outcomes as the course study results

Competence code	Competence descriptor	Previous courses/modules*	Subsequent courses/modules*
GPC-1	Ability to apply the fundamental knowledge gained in the field of mathematical and (or) natural sciences and use it in professional activities	Mathematical analysis, Linear algebra and analytic geometry, Partial differential equations, Functional analysis, Complex analysis, Optimization and convex analysis	State examination, Research seminar, Research work, Function spaces (elective)
PC-1	Ability to determine Generic forms and rules of a separate subject area	Mathematical analysis, Linear algebra and analytic geometry, Functional analysis, Complex analysis	State examination, Research seminar

* To be filled in according to the competence matrix of the higher education programme.

4. COURSE WORKLOAD AND ACADEMIC ACTIVITIES

Possible wording

1) The total workload of the course amounts to 4 credits.

*Table 4.1. Types of academic activities during the periods of higher education programme mastering (**full-time training**)**

Type of academic activities		Total academic hours	Semesters/training modules			
			5	6	7	8
Contact academic hours		*68*			*68*	
including:						
Lectures (LC)		34			34	
Lab work (LW)						
Seminars (workshops/tutorials) (S)		34			34	
Self-studies		*49*			*49*	
Evaluation and assessment (exam/passing/failing grade)		*27*			*27*	
Course workload	academic hours_	**144**			**144**	
	credits	**4**			**4**	

5. COURSE CONTENTS

Table 5.1. Course contents and academic activities types

Course module title	Course module contents (topics)	Academic activities types
Chapter 1: Foundations of topological vector spaces and construction of the	Topic 1.1. Linear topological spaces and their basic properties. Balanced, convex and absorbing sets in linear topological spaces. Locally convex	LC, S

Course module title	Course module contents (topics)	Academic activities types
spaces of test functions and associated distribution spaces	spaces and their equivalent description in terms of a polynormed topology. Examples: topology of the pointwise convergence, strong operator topology, weak topology, weak *-topology, weak operator topology.	
	Topic 1.2. Metrization of the polynormed space with a countable family of pseudonorms. Frechet spaces. Pseudonormability of the polynormed space with a finite family of seminorms. The support of a function. Notion of a continuous embedding between linear topological spaces. Continuous embeddings between (pseudo)normed spaces. Spaces of test functions: definition, embeddings and examples. Classical bump functions and their modifications. Examples of the distributions obtained from the concepts of "valeur principal" and "partie finie". Spaces of test functions as multiplicative algebras. Metrization properties for the spaces of test functions. Density of the space of compactly supported smooth functions in the Schwartz space and in the space of all smooth functions.	LC, S
	Topic 1.3. The dual spaces of distributions: distributions with a compact support, Schwartz (tempered) distributions and Generic distributions. Topology and characterization of convergence in classical distribution spaces. Embeddings between classical distribution spaces. Locally integrable functions: examples and basic properties. Regular distributions generated by locally integrable functions. Examples of singular distributions: delta-function, Generic Radon measures. Support of a distribution.	LC, S
Chapter 2: Main operations and constructions on distribution spaces	Topic 2.1. Differentiation of a distribution: definition and relation with the notions of a Genericized Sobolev and classical derivatives. Examples: derivatives of the Heavyside function and other classical regular and singular distributions. Algebraic rules of the distributional differentiation. Finite order distributions: examples and basic properties.	LC, S
	Topic 2.2. Multiplication of a distribution by a smooth function: definition and basic properties. The relationship between the multiplication operator and differentiation. The Genericized Leibniz formula for the product of a smooth function and a distribution.	LC, S
	Topic 2.3. Convolution of a pair of functions: definition, commutativity, associativity,	LC, S

Course module title	Course module contents (topics)	Academic activities types
	distributivity and other basic properties. Genericized Hoelder's inequality and Young's convolutional inequality. The relationship between convolution and differentiation. Convolution of a function and a distribution. Delta-function as a neutral element with respect to convolution. The relationship between convolution and distributional differentiation.	
Chapter 3: The Fourier transform in distribution spaces	Topic 3.1. The Fourier transform in the Schwartz space of rapidly decreasing functions: definition, differentiation properties, continuity. The inverse Fourier transform in the Schwartz space of rapidly decreasing functions and the Fourier inversion theorem for the Schwartz space of test functions. The Fourier transform as a homeomorphism of the Schwartz space of rapidly decreasing functions onto itself. Plancherel-Schwartz theorem on the extension of the Fourier transform. The Fourier transform of a convolution of two functions. The Fourier transform of a pointwise product of two functions.	LC, S
	Topic 3.2. The Fourier transform and the inverse Fourier transform as homeomorphisms of a dual Schwartz space of tempered distributions. The Fourier transforms of a distributional derivative. The Fourier transform of a convolution of a function and a distribution. The Fourier transform of a distribution multiplied by a function. Paley-Wiener theorem on the analyticity of functions with the compactly supported Fourier transforms. Paley-Wiener-Schwartz theorem on the analytic regularity of tempered distributions with the compactly supported Fourier transform.	LC, S
Chapter 4: Sobolev spaces and distributions	Topic 4.1. Definition of the classical Sobolev function spaces. Definition of the Bessel potential spaces as subspaces of a dual Schwartz space. Completeness of the classical Sobolev spaces. Inner product on the classical Sobolev space and the Bessel potential space in case of a Hoelder constant 2. Friedrichs mollifiers and approximation by smooth functions.	LC, S
	Topic 4.2. Existence of a natural isomorphism between the Bessel potential spaces with nonnegative integer smoothness indices and the classical Sobolev spaces. Formulation of the duality theorem for the Bessel potential spaces. Formulation of the Sobolev embedding theorem. Basics of the complex interpolation in the scales	LC

Course module title	Course module contents (topics)	Academic activities types
	of the classical Sobolev spaces and of the Bessel potential spaces.	

* - to be filled in only for **full** -time training: *LC - lectures; LW - lab work; S - seminars.*

6. CLASSROOM EQUIPMENT AND TECHNOLOGY SUPPORT REQUIREMENTS

Table 6.1. Classroom equipment and technology support requirements

Type of academic activities	**Classroom equipment**	**Specialised educational / laboratory equipment, software, and materials for course study** (if necessary)
Lecture	A lecture hall for lecture-type classes, equipped with a set of specialised furniture; board (screen) and technical means of multimedia presentations.	
Seminar	A classroom for conducting seminars, group and individual consultations, current and mid-term assessment; equipped with a set of specialised furniture and technical means for multimedia presentations.	Moving blackboards and equipment for multimedia presentation are advisable
Self-studies	A classroom for independent work of students (can be used for seminars and consultations), equipped with a set of specialised furniture and computers with access to the electronic information and educational environment.	

* The premises for students' self-studies are subject to **MANDATORY** mention

7. RESOURCES RECOMMENDED FOR COURSE STUDY

Main readings:

1. G. Grubb. Distributions and Operators. Graduate Texts in Mathematics (Book 252), Spinger, 2009.
2. F.G. Friedlander, M. Joshi. Introduction to the Theory of Distributions. Cambridge University Press, Cambridge, 1988.
3. G. van Dijk. Generic functions. De Gruyter Graduate Lectures, De Gruyter, Berlin / Boston, 2013
4. A.Ya. Helesmkii. Lectures and Exercises on Functional Analysis. American Mathematical Society, Providence, RI, 2006.

Additional readings:

1. S.L. Sobolev. Some Applications of Functional Analysis to Mathematical Physics. American Mathematical Society, Providence, RI, 1991.

2. I.M. Gel'fand, G.E. Shilov. Genericized Functions, Vol. 1: Properties and Operations. American Mathematical Society, Providence, RI, 1964.
3. I.M. Gel'fand, G.E. Shilov. Genericized Functions, Vol. 2: Spaces of Fundamental and Genericized Functions. American Mathematical Society, Providence, RI, 1968.
4. D. Haroske, H. Triebel. Distributions, Sobolev Spaces, Elliptic Equations. European Mathematical Society, Zuerich, 2008.
5. S.G. Georgiev. Theory of Distributions. Springer, 2015.
6. J.J. Duistermaat, J.A.C. Kolk. Distributions: Theory and Applications. Springer, 2010.
7. H. Triebel. Interpolation Theory, Function Spaces, Differential Operators. North-Holland, Amsterdam, 1978.
8. R. Adams, J. J.F. Fournier. Sobolev Spaces. 2nd edition. Elsevier, Oxford, 2003.
9. A.P. Robertson, W. Robertson. Topological Vector Spaces. 2nd edition. Cambridge University Press, Cambridge, 1973.
10. D.E. Edmunds, W.D. Evans. Spectral Theory and Differential Operators. Clarendon Press, Oxford, 1987.

Internet sources

1. Electronic libraries (EL) of RUDN University and other institutions, to which university students have access on the basis of concluded agreements:

- RUDN Electronic Library System (RUDN ELS) http://lib.rudn.ru/MegaPro/Web
- EL "University Library Online" http://www.biblioclub.ru
- EL "Yurayt" http://www.biblio-online.ru
- EL "Student Consultant" www.studentlibrary.ru
- EL "Lan" http://e.lanbook.com/
- EL "Trinity Bridge"
- ……….

2.Databases and search engines:

- Mathematics Stack Exchange (Q&A for people studying mathematics at any level and professionals in related fields): http://math.stackexchange,com
- ProofWiki (Online compendium of mathematical proofs): https://proofwiki.org/wiki/Category:Distributions
- NCatLab (articles on distributions and related notions from the categorial viewpoint): https://ncatlab.org/nlab/show/distribution

*Training toolkit for self- studies to master the course *:*

1. The set of lectures on the course "Generic functions"

* The training toolkit for self- studies to master the course is placed on the course page in the university telecommunication training and information system under the set procedure.

8. ASSESSMENT TOOLKIT AND GRADING SYSTEM* FOR EVALUATION OF STUDENTS' COMPETENCES LEVEL UPON COURSE COMPLETION

The assessment toolkit and the grading system* to evaluate the competences formation level (competences in part) upon the course study completion are specified in the Appendix to the course syllabus.

* The assessment toolkit and the grading system are formed on the basis of the requirements of the relevant local normative act of RUDN University (regulations / order).

DEVELOPERS:

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