

*Federal state autonomous educational institution of higher education  
Peoples' Friendship University of Russia*

*Faculty of science*  
Recommended by MSSN

**DISCIPLINE PROGRAM**

**Variational problems**

**Recommended for the study direction**

**01.06.01 « Mathematics and mechanics»**

## 1. Goals and objectives of the discipline

The main goal of the discipline "Variational Problems" is to master the basic methods of modern calculus of variations, which form the basis for expanding the scope of direct variational methods and symmetric analysis. Among them are methods for constructing variational principles for equations with non-potential operators, the Ritz method, the least squares method, variational symmetries, etc. Development of logical thinking.

## 2. The place of the discipline in the structure of HEP:

The discipline "Variational problems" refers to the variational part of block 1 of the curriculum.

Table 1 shows the previous and subsequent disciplines aimed at forming the competencies of the discipline in accordance with the matrix of competencies of the Higher Educational Institution of HEP.

Table 1

### Previous and subsequent disciplines aimed at the formation of competencies

№ п/п	Code and name of the competence	Previous disciplines	Subsequent disciplines (groups of disciplines)
Professional competencies			
	<p>PC-2 knowledge of the theory of functional spaces (Lebesgue spaces, Sobolev spaces, Nikolsky-Besov spaces, etc.) and its applications to the theory of partial differential equations, including the solvability and smoothness of solutions to boundary value problems for elliptic equations, in the field of nonlinear analysis, the theory of extreme problems, and optimal control</p> <p>PC-3 ability to formulate the research task and the ways of its implementation, to summarize the results and formulate appropriate conclusions, to understand the practical aspects of the theoretical results.</p>	-	<p>General theory of functional differential equations;                      Nonlinear partial differential equations;                      Variational analysis of differential equations;                      Mathematical methods in Economics.</p>
Universal competencies			
	<p>UC-1 ability to critically analyze and evaluate modern scientific achievements, generate new ideas in solving research and practical prob-</p>	-	<p>General theory of functional differential equations;                      Nonlinear partial differential equations;                      Variational analysis of differential equations;                      Mathematical methods in</p>

<p>blems including interdisciplinary fields;  UC-2 ability to design and implement comprehensive research including interdisciplinary research based on a holistic systematic scientific worldview using knowledge in the field of history and philosophy of science;  UC-3 willingness to participate in the work of Russian and international research teams to solve scientific and educational problems;  UC-5 ability to plan and solve problems of their own professional and personal development.</p>		Economics.
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### 3. Requirements for the results of mastering the discipline:

The process of mastering the discipline is aimed at the formation of the following competencies: UC-1, UC -2, UC -3, UC -5, PC -2, 3/

As a result of studying the discipline, the student must:

**Know:** Statements of the main boundary value problems of mathematical physics, the concepts of symmetric, positive and positive definite operators, the Ritz method, the method of least squares, the concept of a potential operator, the conditions of potentiality, the concept of a minimizing sequence and methods of its construction, the concept of a Hamiltonian operator, elements of Lie algebra, variational symmetries.

**Be able to:** Solve problems in all sections of the course at the level set by the attached examples.

**Possess:** The beginnings of each of the sections presented in the course.

### 4. The scope of the discipline and types of academic work

The total labor intensity of the course " Variational Problems " is 2 credits.

Type of study work	Total hours	Semesters			
<b>Classes (total)</b>		1		3	
Including:	-	-		-	-
<i>Lectures</i>	6	6			
<i>Practicum (P)</i>	12	12			
<i>Seminars (S)</i>					

<i>Laboratory work (LW)</i>				
<b>Individual work (total)</b>	54	54		
Total labor intensity	72	72		
	2	2		

## 5. Content of the discipline

### 5.1. Content of the discipline sections

№ п/п	Units	Topics
1.	A direct variation problem	Gato derivative and differential. Extreme points of the functional. Bilinear and quadratic functionals. Euler-Lagrange Equations. Symmetric, positive, and positive definite operators. The Ritz Method.
2.	Potential operators and variational symmetries	Potential operators. The criterion of potentiality and the formula for constructing the functional. Conditions of potentiality of systems of ordinary differential equations with derivatives of the first order. Algebraic and geometric values of potentiality conditions. Classical Hamiltonian systems and their potentiality. Conditions of potentiality of systems of differential and integro-differential equations with partial derivatives. Variational symmetries.
3.	Inverse problems of the variation calculations	Kirchhoff equations and their universal value Statement of the classical problem for the General equation and systems with derivatives of the second order. Construction of variational principles for dissipative problems. Examples.
4.	Inverse problems of the variation calculations for partial differential equation	Non-existence of semi-bounded solutions of problems for some classical equations. Non-Eulerian functional classes. Constructive problem solving for a boundary value problem with a parabolic operator. Satisfaction of the criterion of generalized potentiality due to the choice of bilinear form and variational factors. E. Tanti's scheme of problem solving for equations with nonlinear non-potential operator. problems for an evolutionary operator equation with a first-order derivative in time. Hamiltonian operators. An example of the Korteweg-de Vries equation.

### 5.2. Sections and classes

№ п/п	Units	Lect.	Practicum and laboratory work	IW	Total
1.	A direct variation problem Potential operators and variational symmetries	1	2	12	15

2.	Inverse problems of the variation calculations	1	2	12	15
3.	A direct variation problem Potential operators and variational symmetries	2	4	15	21
4.	Inverse problems of the variation calculations	2	4	15	21

**6. Laboratory practice** – not provided.

**7. Practicum (Seminars)**

№ п/п	Unit №	Practicum units (seminars)	Total labor intensity (hour)
1	1	The Euler-Lagrange equations. Symmetric, positive, and positive definite operators. The Ritz method.	2
2	2	Conditions for the potentiality of systems of differential and integro-differential partial differential equations. Variational symmetries.	2
3	3	ODS for ODE systems with first-order derivatives. ODS for ODE systems with second-order derivatives.	2
4	3	Construction of variational principles for dissipative problems.	2
5	4	Non-Eulerian classes of functionals. Satisfaction of the potentiality criterion by choosing a bilinear form and variational multipliers.	2
6	4	ZVI for an evolutionary operator equation with a first-order time derivative and their relation to Hamilton	2

## **8. Material and technical support of the discipline:**

Auditorium 495a, 398, 509 Ordzhonikidze str., 3, RUDN, group classrooms Ordzhonikidze str., 3, RUDN on the 3rd, 4th and 5th floors, display classes, laboratories (rooms 510 and 424).

## **9. Information support of the discipline:**

Only licensed software installed in the RUDN is used:

- Microsoft Office software package;
- multimedia equipment and personal computers;
- full-text databases and resources accessed from the RUDN network;
- electronic library of the RFBR <http://www.rfbr.ru/rffi/ru/library>

## **10. Educational and methodological support of the discipline:**

### **a) basic literature:**

1. Courant R., Gilbert D. Methods of mathematical physics, M.-L., 1951, vol. 1, 2.
2. Rectoris K. Variational methods in mathematical physics and Engineering, M., 1985.
3. Gaevsky H., Grecher K., Zacharias K. Nonlinear operator equations and operator differential equations, M., 1985.
4. Filippov V. M. Variational principles for non-potential operators, M., 1985.

### **b) additional literature:**

1. Marchuk G. I., Agoshkov V. I. Introduction to projection-grid methods, M., 1981.
2. Mikhlin S. G. The problem of the minimum of a quadratic functional, M., 1952.
3. Vainberg M. M. Functional analysis, M., 1979.
4. Filippov V. M., Savchin V. M., Shorokhov S

### **в) databases, information and reference systems and search engines**

1. Higher Attestation Commission RF <http://vak.ed.gov.ru>
2. RSL Electronic Library <http://www.rsl.ru/>
3. RUDN Library <http://lib.rudn.ru/>
4. Science Direct <http://www.sciencedirect.com> Description: The resource contains a collection of scientific, technical full-text and bibliographic information. The multidisciplinary database includes scientific journals in the exact and technical fields of science.
5. EBSCO <http://search.ebscohost.com>, Academic Search Premier (a database of complex topics, containing information on the humanities and natural sciences).
6. Oxford University Press <http://www3.oup.co.uk/jnls>. Journals in the exact and technical sciences of Oxford University Press presented in the collection HSS
7. Sage Publications <http://online.sagepub.com> . The Sage publication database includes journals in various fields of knowledge: Sage\_STM – more than 100 journals in the field of natural sciences, engineering.
8. Springer/Kluwer <http://www.springerlink.com>. Journals and books publishing houses
9. Springer/Kluwer cover various fields of knowledge and are divided into subject categories.
10. Tailor & Francis <http://www.informaworld.com> . The collection of journals includes more than 1000 titles in all fields of knowledge.
11. American Mathematical Society <http://www.ams.org/> A resource of the American Mathematical Society.

12. European Mathematical Society <http://www.euro-math-soc.eu/> Resource of the European Mathematical Society.
13. Portal to Mathematics Publications <http://www.emis.de/projects/EULER/>
14. Catalog of mathematical Internet resources <http://www.mathtree.ru/>
15. Zentralblatt MATH (zbMATH) <https://zbmath.org>
16. All-Russian mathematical portal mathnet.ru
17. Web of Science <http://www.isiknowledge.com>
18. Resources of the Institute of Scientific Information on Social Sciences of the Russian Academy of Sciences <http://elibrary.ru>.
19. University Information System RUSSIA. <http://www.cir.ru/index.jsp>.
20. GOST standards system for information, library and publishing <http://www.ifap.ru/library/gost/sibid.htm>.
21. RUDN Electronic Library <http://www.rsl.ru/>

## r) periodicals

Algebra i analiz, Diskretnaya matematika, Zhurnal vychislitel'noj matematiki i matematicheskoy fiziki, Izvestiya Rossijskoj akademii nauk. Seriya matematicheskaya, Matematicheskie zametki Matematicheskij sbornik, Matematicheskoe modelirovanie, Teoreticheskaya i matematicheskaya fizika, Teoriya veroyatnostej i ee primeneniya, Uspekhi matematicheskikh nauk, Funkcional'nyj analiz i ego prilozheniya, Trudy Matematicheskogo instituta im. V. A. Steklova, Sovremennye problemy matematiki, Vychislitel'nye metody i programmirovaniye, Trudy seminarov imeni I. G. Petrovskogo, Uchyonye zapiski Moskovskogo gosudarstvennogo universiteta Fundamental'naya i prikladnaya matematika, Review of Modern Physics, Annual Review of Astronomy and Astrophysics, Annual Review of Biochemistry, Chemical Reviews Nature Physics, Annual Review of Condensed Matter Physics, Annals of Mathematics, Journal of the American Mathematical Society, Acta Mathematica, Communications on Pure and Applied Mathematics Swarm and Evolutionary Computation Geometric and Functional Analysis Formal Aspects of Computing, Discrete Mathematics, Theory of Computing Systems Reports on Progress in Physics New Journal of Physics.

## 11. Methodological guidelines for students on the development of the discipline

The format of the seminar is the presentation of the key ideas of textbook sources - texts on the course. Specifically, the presentation looks like this: the student chooses one of the key ideas of the text discussed at the seminar, prepares his understanding and critical assessment in the form of theses (on 1-1.5 pages), then all this is presented and defended at the seminar. Abstracts are reproduced in advance and distributed to all participants of the seminar.

It is supposed to write an abstract on a topic agreed with the teacher. The volume of the abstract – no more than 15 thousand characters with spaces. The translation of an article by a foreign author, together with a detailed critical and analytical assessment of it, is also accepted as an abstract. Coordination of both the author and the text with the teacher is mandatory.

At the end of the semester an exam in the form of an essay on one of the units proposed (to choose from) by the teacher. After the interview, the final grade is issued. The results are determined by conducting an interim attestation with grades "excellent", "good", "satisfactory", "unsatisfactory" and in the ECTS system (A, B, C, E). The basis for their placement is the point-rating system adopted in the RUDN.

## 12. The fund of evaluation funds for conducting intermediate certification of students in the discipline (module)

Materials for assessing the level of development of educational material of the discipline "Variational problems" (evaluation materials), which include a list of competencies indicating the stages of their formation, a description of indicators and criteria for evaluating competencies at various stages of their formation, a description of assessment scales, standard control tasks or other materials necessary for evaluating knowledge, skills, skills and (or) experience of activities that characterize the stages of competence formation in the process of mastering the educational program, methodological materials defining the procedures for evaluating knowledge, skills, skills and (or) experience activities that characterize the stages of competence formation are fully developed and are available to students on the discipline page in the TUIS PFUR.

The program is compiled in accordance with the requirements of the ES HE PFUR.

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