Federal State Autonomous Educational Institution of Higher Education
Peoples' Friendship University of Russia named after Patrice Lumumba

Academy of Engineering
(name of the main educational unit (MEU) that developed the educational program of higher education)

WORKING PROGRAM OF THE DISCIPLINE

OPTIMAL CONTROL METHODS
(name of discipline/module)

Recommended for the field of study/specialty:

27.03.04 CONTROL IN TECHNICAL SYSTEMS
(code and name of the field of study/specialty)

The discipline is mastered within the framework of the implementation of the main professional educational program of higher education (EP HE):

DATA SCIENCE AND SPACE SYSTEMS
(name (profile/specialization) of the educational institution of higher education)

2026

1. THE GOAL OF MASTERING THE DISCIPLINE

The course "Optimal Control Methods" is part of the "Data Science and Space Systems" bachelor's program, major 27.03.04 "Control in Technical Systems," and is studied in semesters 6 and 7 of years 3 and 4. The course is offered by the Department of Mechanics and Control Processes. It consists of three sections and 23 topics and focuses on the fundamental principles of L.S. Pontryagin's maximum principle, dynamic programming, and numerical methods of optimal control. It also explores the main methods for solving typical problems and introduces their application in professional activities.

The purpose of mastering the discipline is to develop fundamental knowledge and skills in applying problem-solving methods necessary for professional activity, and to improve the general level of students' literacy in management methods.

2. REQUIREMENTS FOR THE RESULTS OF MASTERING THE DISCIPLINE

Mastering the course "Optimal Control Methods" aimed at developing the following competencies (parts of competencies) in students:

Table 2.1. List of competencies developed in students while mastering the discipline (results of mastering the discipline)

Cipher	Competence	Indicators of Competency Achievement (within this discipline)
UC-12	Able to search for relevant sources of information and data, perceive, analyze, memorize, and transmit information using digital tools, as well as algorithms when working with data obtained from various sources in order to effectively use the information obtained to solve problems; evaluate information, its reliability, and draw logical conclusions based on incoming information and data	UC-12.1 Searches for the necessary sources of information and data, perceives, analyzes, remembers and transmits information using digital means, as well as with the help of algorithms when working with data obtained from various sources in order to effectively use the information received to solve problems; UC-12.2 Conducts an assessment of information, its reliability, builds logical conclusions based on incoming information and data;
GPC-11	Able to understand the principles of operation of modern information technologies and use them to solve professional problems	GPC-11.1 Knows digital methods and technologies used in professional activities; GPC-11.2 Able to apply digital methods and technologies in professional activities to study and model objects of professional activity, analyze data, and present information; GPC-11.3 Confidently uses digital methods and technologies in professional activities (in the field of management in technical systems) for: studying and modeling objects of professional activity, data analysis, presentation of information;
GPC-3	Able to use fundamental knowledge to solve basic control problems in technical systems in order to improve in professional activities	GPC-3.1 Knows the theoretical foundations and principles of mathematical modeling; GPC-3.2 Able to develop and use methods of mathematical modeling, information technologies to solve problems of applied mathematics; GPC-3.3 Possesses practical skills in solving problems of applied mathematics, methods of mathematical modeling, information technologies and the basics of their use in professional activities, skills of professional thinking and an arsenal of methods and approaches necessary for the adequate use of methods of modern mathematics in theoretical and applied problems;
GPC-8	Capable of setting up measuring and control equipment and sys-	GPC-8.1 Knows the parameters and characteristics of measuring and control equipment and complexes;

Cipher	Competence	Indicators of Competency Achievement (within this discipline)
	tems, and performing their routine maintenance	GPC-8.2 Can perform routine maintenance of measuring and control equipment and complexes; GPC-8.3 Ensures the adjustment of measuring and control equipment and complexes and their routine maintenance;
GPC-9	Able to perform experiments using given methods and process the results using modern information technologies and technical means	GPC-9.1 Knows modern information technologies and technical means; GPC-9.2 Able to apply modern information technologies and technical means to process experimental results; GPC-9.3 Proficient in modern information technologies and technical means for performing experiments and processing results;
PC-1	Capable of collecting, processing and interpreting modern scientific research data necessary to draw conclusions on relevant scientific research, including Earth remote sensing data	PC-1.1 Knows modern methods of collecting, processing and interpreting data from modern scientific research necessary for drawing conclusions on relevant scientific research; PC-1.2 Able to apply modern methods and tools for processing and interpreting scientific research data; PC-1.3 Possesses the basic skills of collecting, processing and interpreting data from modern scientific research, necessary for drawing conclusions on relevant scientific research;
PC-4	Able to formulate, analyze and solve engineering problems in the field of ballistics, motion mechanics and spacecraft motion control based on professional knowledge	PC-4.1 Knows the basic concepts and basic algorithms for solving problems in the field of ballistics, motion mechanics and motion control based on automated and automatic systems; PC-4.2 Able to solve engineering problems of an analytical nature in the field of ballistics, motion mechanics and spacecraft motion control based on professional knowledge; PC-4.3 Possesses skills in using mathematical methods for processing information obtained as a result of experimental studies, basic methods for analyzing the mechanics of motion and controlling the motion of spacecraft based on standard methods and software packages;

3. PLACE OF THE DISCIPLINE IN THE STRUCTURE OF THE EDUCATIONAL EDUCATIONAL INSTITUTION

Course "Optimal Control Methods" refers to the mandatory part of block 1 “Disciplines (modules)” of the educational program of higher education.

As part of the higher education program, students also master other disciplines and/or practices that contribute to the achievement of the planned results of mastering the discipline "Methods of Optimal Control".

Table 3.1. List of components of the educational program of higher education that contribute to the achievement of the planned results of mastering the discipline

Cipher	Name of competence	Previous courses/modules, practical training*	Subsequent disciplines/modules, practices*
UC-12	Able to search for relevant sources of information and data, perceive, analyze, memorize, and transmit information using digital tools, as well as algorithms when working with data obtained from various sources in order to effectively use the information obtained to solve problems; evaluate information, its reliability,	Research work / Scientific research work; Automatic Control Theory; Analysis of Geoinformation Data; *Fundamentals of Information Security and Cyber Resilience***; *Fundamentals of Information Security and Cyber Resilience***;	Technological Training; Undergraduate Training;

Cipher	Name of competence	Previous courses/modules, practical training*	Subsequent disciplines/modules, practices*
	and draw logical conclusions based on incoming information and data		
GPC-3	Able to use fundamental knowledge to solve basic control problems in technical systems in order to improve in professional activities	Research work / Scientific research work; Mathematical analysis; Space Flight Mechanics; Theoretical Mechanics; Numerical Methods; Automatic Control Theory; Algebra and Geometry; Theory of Probability and Mathematical Statistics; Differential equations; Complex analysis; Analysis of Geoinformation Data;	Technological Training; Undergraduate Training;
GPC-8	Capable of setting up measuring and control equipment and systems, and performing their routine maintenance		Undergraduate Training;
GPC-9	Able to perform experiments using given methods and process the results using modern information technologies and technical means	Computer Science and Programming; Analysis of Geoinformation Data; Basic Military Training. Life Safety; Introduction to Computing Science;	Undergraduate Training; Technological Training;
GPC-11	Able to understand the principles of operation of modern information technologies and use them to solve professional problems	Space Flight Mechanics;	Undergraduate Training; Technological Training;
PC-1	Capable of collecting, processing and interpreting modern scientific research data necessary to draw conclusions on relevant scientific research, including Earth remote sensing data	Space Flight Mechanics; Numerical Methods; Automatic Control Theory; Computer Science and Programming; *Discrete Mathematics***; *Discrete Mathematics***; Analysis of Geoinformation Data; Introduction to Computing Science; Research work / Scientific research work;	Technological Training; Undergraduate Training;
PC-4	Able to formulate, analyze and solve engineering problems in the field of ballistics, motion mechanics and spacecraft motion control based on professional knowledge	Research work / Scientific research work; Space Flight Mechanics; Theoretical Mechanics;	Technological Training; Undergraduate Training;

* - filled in accordance with the competency matrix and the SUP EP HE
** - elective courses/practices

4. SCOPE OF THE DISCIPLINE AND TYPES OF EDUCATIONAL WORK

The total workload of the discipline “Methods of optimal control” is 10 credit units.

Table 4.1. Types of educational work by periods of mastering the educational program of higher education for full-time education.

Type of academic work	**TOTAL,** academic hours		**Semester(s)**	
			6	**7**
Contact work, academic hours	*162*		*72*	*90*
Lectures (LC)	72		36	36
Laboratory work (LW)	72		36	36
Practical/seminar classes (SC)	18		0	18
Independent work of students, academic hours	*144*		*81*	*63*
Control (exam/test with assessment), academic hours	*54*		*27*	*27*
Total complexity of the discipline	**academic hours**	***360***	*180*	*180*
	credit	***10***	*5*	*5*

The total workload of the discipline “Methods of optimal control” is 10 credit units.

Table 4.2. Types of educational work by periods of mastering the educational program of higher education for full-time education.

Type of academic work	**TOTAL,** academic hours		**Semester(s)**	
			6	**7**
Contact work, academic hours	*162*		*72*	*90*
Lectures (LC)	72		36	36
Laboratory work (LW)	72		36	36
Practical/seminar classes (SC)	18		0	18
Independent work of students, academic hours	*144*		*81*	*63*
Control (exam/test with assessment), academic hours	*54*		*27*	*27*
Total complexity of the discipline	**academic hours**	***360***	*180*	*180*
	credit	***10***	*5*	*5*

The total workload of the discipline “Methods of optimal control” is 10 credit units.

Table 4.3. Types of educational work by periods of mastering the educational program of higher education for full-time education.

Type of academic work	**TOTAL,** academic hours		**Semester(s)**	
			6	**7**
Contact work, academic hours	*162*		*72*	*90*
Lectures (LC)	72		36	36
Laboratory work (LW)	72		36	36
Practical/seminar classes (SC)	18		0	18
Independent work of students, academic hours	*144*		*81*	*63*
Control (exam/test with assessment), academic hours	*54*		*27*	*27*
Total complexity of the discipline	**academic hours**	***360***	*180*	*180*
	credit	***10***	*5*	*5*

5. CONTENT OF THE DISCIPLINE

Table 5.1. Content of the discipline (module) by types of academic work

Section number	Name of the discipline section	Topic Title		Topic Contents	Type of academic work*
Section 1	Optimal control theory. L.S. Pontryagin's maximum principle.	1.1	Statement of optimal control problems. Basic concepts. Examples of optimal control problems.	The subject of optimal control theory. Concepts of a control system, phase coordinates, control functions, and admissible control. Performance criteria (target functionals). Control constraints and phase coordinates. Examples: spacecraft control, optimal response, and energy cost minimization.	LC, SC
		1.2	Problems with a free right endpoint. Formula for incrementing the functional.	Problem statement where the final state of the system is not fixed. Concept of functional increment with control variation. Derivation of a formula for the increment, allowing one to evaluate how a change in control affects the value of the objective functional.	SC
		1.3	L.S. Pontryagin's maximum principle for problems with a free right end. Formulation and proof.	Introduction of adjoint variables and the Hamiltonian function. Formulation of the maximum principle: for optimal control, the Hamiltonian function attains its maximum with respect to control at every instant. Key steps of the proof using the functional increment formula.	LC, LW
		1.4	Linear problems with a free right endpoint. The maximum principle as a necessary and sufficient condition.	Characteristics of linear control systems. For linear problems with a convex domain of admissible controls and a convex functional, the maximum principle becomes not only a necessary but also a sufficient condition for optimality.	LC, LW
		1.5	Formulation of the maximum principle for various classes of optimal control problems: a) two-point problems; b) optimal response problem; c) problems with boundary conditions, transversality conditions; d) autonomous and non-autonomous systems; d) problems with fixed and non-fixed completion time of the process; e) problems with integral and terminal functional; g) problems with parameters.	Generalization of the maximum principle: for two-point problems with fixed start and end; for the problem of optimal response (minimization of the transition time); for problems with boundary conditions (transversality conditions at the free ends); for autonomous (not explicitly dependent on time) and non-autonomous systems.	LC, SC
		1.6	Examples of optimal control problems. The problem of time-response.	Case studies. Optimal response problem for a linear system: determining the control law that ensures the minimum transition time from the initial to the final point. Analysis of the relay nature of optimal control.	LC, LW
		1.7	The concept of optimal control synthesis.	Synthesis as the construction of optimal feedback control: the	LC, SC

Section number	Name of the discipline section	Topic Title		Topic Contents	Type of academic work*
				dependence of control on the current state of the system. The difference between synthesis and program control. The importance of synthesis for practical implementation.	
		1.8	The relationship of the maximum principle to the classical calculus of variations. Derivation of the Euler equation and the Legendre-Clebsch conditions from the maximum principle. The Jacobi condition.	Transition from the maximum principle to classical optimality conditions. Derivation of the Euler equation from the maximum principle. Derivation of the Legendre-Clebsch conditions (analogous to the convexity condition). The concept of the Jacobi condition and its relationship to conjugate variables.	LC, LW
Section 2	Dynamic programming	2.1	Controlled multi-step processes. The optimality principle.	The concept of multi-step control processes. Discrete systems. Bellman's optimality principle: optimal behavior in the remaining segment is independent of the previous history and is determined only by the current state.	LC, SC
		2.2	Dynamic programming method for multi-step control processes.	Recursive procedure for constructing optimal control. Movement from the end of the process to the beginning. Bellman function (conditional optimum). Computational scheme of the method.	LC, LW
		2.3	Dynamic programming method for optimal control problems.	Extending the ideas of dynamic programming to continuous processes. Transition from multi-step to continuous processes. Limit transition.	LC, SC
		2.4	Bellman differential equation. Statement of problems for the Bellman equation. Examples.	Derivation of the partial differential equation for the value function (Bellman function). The Hamilton-Jacobi-Bellman equation. Boundary conditions for the Bellman equation. Examples of solutions to simple problems.	LC, LW
		2.5	The relationship between the dynamic programming method and the maximum principle. Deriving transversality conditions using the dynamic programming method.	Comparison of two approaches to optimal control. Derivation of transversality conditions (free-end conditions) using dynamic programming. Equivalence of the maximum principle and the Bellman equation under certain smoothness conditions.	LC, SC
		2.6	Linear control systems with quadratic functional. Construction of optimal control synthesis.	Statement of a linear-quadratic problem. Features: linear dynamics and quadratic performance criterion. Construction of a synthesis in the form of linear feedback. Solution of the Riccati equation.	LC, LW
Section 3	Numerical methods of optimal control	3.1	Numerical methods based on reducing optimal control problems to boundary value problems using the maximum principle.	Using the maximum principle to obtain a two-point boundary value problem for a system of differential equations (phase and adjoint). Reducing the optimization problem to solving a boundary value problem.	LC, SC
		3.2	Using methods for solving systems of algebraic equations to solve boundary value problems. Newton's method and its	Discretization of a boundary value problem. Reduction to a system of nonlinear algebraic equations. Newton's method for solving such systems. Modifications of the method (the secant method, quasi-	LC, SC

Section number	Name of the discipline section	Topic Title		Topic Contents	Type of academic work*
			modifications.	Newton methods) to improve stability.	
		3.3	Numerical methods for minimizing functions of several variables. Concepts of linear and nonlinear programming. Gradient method. Penalty function method.	Problems of unconstrained and constrained minimization. Linear and nonlinear programming. Gradient method: the idea of moving in the direction of the fastest possible decrease of a function. Penalty function method for accounting for constraints: adding a penalty to the objective function for violating constraints.	LC, LW
		3.4	Numerical methods based on varying control functions. Gradient method in control space. Accounting for constraints on control functions. Accounting for boundary conditions and phase constraints using the penalty function method. Accounting for boundary conditions using the gradient projection method.	Direct methods: varying the control as a function of time. Calculating the gradient of a functional with respect to the control. Gradient descent in control space. Taking into account constraints on control functions (gradient projection). Taking into account boundary conditions and phase constraints using penalty functions.	LC, LW
		3.5	The method of successive approximations in the space of control functions. Methods for improving convergence and modifications. Examples.	An iterative process for constructing a control sequence converging to the optimal one. Step selection in the gradient method. Methods for accelerating convergence: conjugate gradient methods, variable metric methods. Application examples.	LC, LW
		3.6	Small parameter method for weakly controlled systems.	Asymptotic methods for systems with small control influence. Expanding the solution into a series in a small parameter. Constructing approximate optimal control.	LC, SC
		3.7	Numerical methods based on varying phase coordinates in space. Dynamic programming. Complete and partial enumeration. The "wandering tube" method.	Approaches based on phase space discretization. Dynamic programming in numerical form: grid methods. The "curse of dimensionality" problem. Complete grid enumeration. Partial enumeration (the "wandering tube" method) for reducing computational complexity.	LC, SC
		3.8	The concept of an elementary operation and methods for constructing it. Constructing an elementary operation for flight dynamics problems.	An elementary operation as a typical step in a numerical method. Techniques for constructing elementary operations for solving boundary value problems. Constructing an elementary operation for flight dynamics problems (e.g., calculating a trajectory over a small segment with constant parameters).	LC, LW
		3.9	Method of local variations. Application of the method of local variations to various variational problems. Variational problems with non-additive functionals. Variational problems in partial derivatives.	The idea of the method: sequential variation of control over individual time intervals. Algorithm of the method of local variations. Application to classical variational problems. Extension to variational problems with non-additive functionals. Application to variational problems in partial derivatives.	LC, LW

* - to be completed only for FULL-TIME education: LC – lectures; LW – laboratory work; SC – practical/seminar classes.

6. LOGISTIC AND TECHNICAL SUPPORT OF DISCIPLINE

Table 6.1. Material and technical support for the discipline

Audience type	Equipment of the auditorium	Specialized educational/laboratory equipment, software and materials for mastering the discipline (if necessary)
Lecture	A lecture hall equipped with specialized furniture, a whiteboard (screen), and multimedia presentation equipment.	Projector
Laboratory	A classroom for laboratory work, individual consultations, ongoing monitoring and midterm assessment, equipped with a set of specialized furniture and equipment.	
Seminar	An auditorium for conducting seminar-type classes, group and individual consultations, ongoing monitoring and midterm assessment, equipped with a set of specialized furniture and technical means for multimedia presentations.	
For independent work	A classroom for independent student work (can be used for seminars and consultations), equipped with a set of specialized furniture and computers with access to the Electronic Information System.	

* - the classroom for independent work of students MUST be indicated!

7. EDUCATIONAL, METHODOLOGICAL AND INFORMATIONAL SUPPORT OF THE DISCIPLINE

Main literature:

1. Pontryagin L.S., Boltyansky V.G., Gamkrelidze R.V., Mishchenko E.F. Mathematical theory of optimal processes. Moscow: Nauka, 1969.
2. Ivanov V.A., Faldin P.V. Theory of optimal automatic control systems. Moscow: Nauka, 1981. 336 p.
3. Roytenberg Ya.N. Automatic control, Moscow: Nauka, 1971. 396 p.
4. Afanasyev V.N., Kolmanovsky V.B., Nosov V.R. Mathematical theory of control systems design. Moscow: Higher School, 2003.

Further reading:

1. Gelfand I.M., Fomin S.V. Calculus of variations. Moscow: Fizmatlit, 1961.
2. Boltyansky V.G. Mathematical methods of optimal control. Moscow: Nauka, 1969.
3. Bellman R., Dreyfus S. Applied problems of dynamic programming. Moscow: Nauka, 1965.
4. Moiseev N.N. Elements of the theory of optimal systems. Moscow: Nauka, 1975.
5. Chernousko F.L., Banichuk N.V. Variational problems of mechanics and control. Numerical methods. Moscow: Nauka, 1973.
6. Chernousko F.L., Akulenko L.D., Sokolov B.N. Control of Oscillations. Moscow: Nauka, 1980.
7. Chernousko F.L. Assessment of the phase state of dynamic systems. Moscow:

Fizmatlit, 1988.
8. Chernousko F.L., Ananevsky I.M., Reshmin S.A. Methods of control of nonlinear mechanical systems. Moscow: Fizmatlit, 2006.
9. Chernousko FL, Ananievski IM, Reshmin SA Control of Nonlinear Dynamical Systems.Methods and Applications. Berlin, Heidelberg: Springer, 2008, 396 p.
10. Lee E.B., Marcus L. Fundamentals of Optimal Control Theory / Translated from English. Moscow: Nauka, 1972. 576 p.
11. Pontryagin L.S. The Maximum Principle. Moscow: Foundation for Mathematical Education and Enlightenment, 1998.

Resources of the information and telecommunications network "Internet":

1. RUDN University Electronic Library System and third-party electronic library systems to which university students have access based on concluded agreements
    - Electronic library system of RUDN - ELS RUDN http://lib.rudn.ru/MegaPro/Web
    - Electronic Library System "University Library Online" http://www.biblioclub.ru
    - EBS Yurayt http://www.biblio-online.ru
    - Electronic Library System "Student Consultant" www.studentlibrary.ru
    - Electronic Library System "Troitsky Bridge"
2. Databases and search engines
    - electronic fund of legal and regulatory documentation http://docs.cntd.ru/
    - Yandex search engine https://www.yandex.ru/
    - Google search engine https://www.google.ru/
    - SCOPUS abstract database http://www.elsevierscience.ru/products/scopus/

Educational and methodological materials for independent work of students in mastering a discipline/module:*

1. Lecture course on the subject "Methods of optimal control".

* - all teaching and methodological materials for independent work of students are posted in accordance with the current procedure on the discipline page in TUIS!

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