Federal State Autonomous Educational Institution of Higher Education
Peoples' Friendship University of Russia named after Patrice Lumumba

Academy of Engineering

(name of the main educational unit (MEU) that developed the educational program of higher education)

WORKING PROGRAM OF THE DISCIPLINE

NUMERICAL METHODS

(name of discipline/module)

Recommended for the field of study/specialty:

27.03.04 CONTROL IN TECHNICAL SYSTEMS

(code and name of the field of study/specialty)

The discipline is mastered within the framework of the implementation of the main professional educational program of higher education (EP HE):

DATA SCIENCE AND SPACE SYSTEMS

(name (profile/specialization) of the educational institution of higher education)

2026

1. THE GOAL OF MASTERING THE DISCIPLINE

The course "Numerical Methods" is part of the bachelor's program "Data Science and Space Systems" (27.03.04 "Control in Technical Systems") and is studied in the fifth semester of the third year. The course is offered by the Department of Mechanics and Control Processes. It consists of 10 sections and 65 topics and focuses on the study of classical numerical methods for solving mathematical problems, including the most effective and important ones from an optimization perspective.

The goal of mastering the discipline is to obtain the necessary knowledge for the implementation of numerical optimization methods in algorithmic programming languages

2. REQUIREMENTS FOR THE RESULTS OF MASTERING THE DISCIPLINE

Mastering the discipline "Numerical Methods" aimed at developing the following competencies (parts of competencies) in students:

Table 2.1. List of competencies developed in students while mastering the discipline (results of mastering the discipline)

Cipher	Competence	Indicators of Competency Achievement (within this discipline)
GPC-2	Able to formulate objectives for professional activity based on knowledge of specialized sections of mathematical and natural science disciplines (modules)	GPC-2.1 Has a command of mathematical methods, programming fundamentals and specialized programming systems for implementing algorithms for solving applied problems; GPC-2.2 Able to select and adapt mathematical methods and software to solve practical problems; GPC-2.3 Possesses skills in developing and implementing algorithms for solving applied problems in the field of professional activity;
GPC-3	Able to use fundamental knowledge to solve basic control problems in technical systems in order to improve in professional activities	GPC-3.1 Knows the theoretical foundations and principles of mathematical modeling; GPC-3.2 Able to develop and use methods of mathematical modeling, information technologies to solve problems of applied mathematics; GPC-3.3 Possesses practical skills in solving problems of applied mathematics, methods of mathematical modeling, information technologies and the basics of their use in professional activities, skills of professional thinking and an arsenal of methods and approaches necessary for the adequate use of methods of modern mathematics in theoretical and applied problems;
PC-1	Capable of collecting, processing and interpreting modern scientific research data necessary to draw conclusions on relevant scientific research, including Earth remote sensing data	PC-1.1 Knows modern methods of collecting, processing and interpreting data from modern scientific research necessary for drawing conclusions on relevant scientific research; PC-1.2 Able to apply modern methods and tools for processing and interpreting scientific research data; PC-1.3 Possesses the basic skills of collecting, processing and interpreting data from modern scientific research, necessary for drawing conclusions on relevant scientific research;

3. PLACE OF THE DISCIPLINE IN THE STRUCTURE OF THE EDUCATIONAL EDUCATIONAL INSTITUTION

Discipline "Numerical Methods" refers to the mandatory part of block 1 “Disciplines (modules)” of the educational program of higher education.

As part of the higher education program, students also master other disciplines and/or practices that contribute to the achievement of the planned results of mastering the discipline "Numerical Methods".

Table 3.1. List of components of the educational program of higher education that contribute to the achievement of the planned results of mastering the discipline

Cipher	**Name of competence**	**Previous courses/modules, practical training***	**Subsequent disciplines/modules, practices***
GPC-2	Able to formulate objectives for professional activity based on knowledge of specialized sections of mathematical and natural science disciplines (modules)	Mathematical analysis; Space Flight Mechanics; Algebra and Geometry; Analysis of Geoinformation Data;	Research work / Scientific research work; Technological Training; Undergraduate Training; Research Work; Space Flight Mechanics; Automatic Control Theory; Equations of mathematical physics; Analysis of Geoinformation Data;
GPC-3	Able to use fundamental knowledge to solve basic control problems in technical systems in order to improve in professional activities	Mathematical analysis; Space Flight Mechanics; Theoretical Mechanics; Algebra and Geometry; Theory of Probability and Mathematical Statistics; Differential equations; Complex analysis; Analysis of Geoinformation Data;	Undergraduate Training; Research work / Scientific research work; Technological Training; Space Flight Mechanics; Automatic Control Theory; Equations of mathematical physics; Optimal Control Methods; Analysis of Geoinformation Data;
PC-1	Capable of collecting, processing and interpreting modern scientific research data necessary to draw conclusions on relevant scientific research, including Earth remote sensing data	Space Flight Mechanics; Computer Science and Programming; *Discrete Mathematics***; *Discrete Mathematics***; Analysis of Geoinformation Data; Introduction to Computing Science;	Space Flight Mechanics; Automatic Control Theory; *Virtual and Augmented Reality Technology***; *Virtual and augmented reality technologies***; Optimal Control Methods; Analysis of Geoinformation Data; Research work / Scientific research work; Technological Training; Undergraduate Training;

* - filled in accordance with the competency matrix and the SUP EP HE
** - elective courses/practices

4. SCOPE OF THE DISCIPLINE AND TYPES OF EDUCATIONAL WORK

The total workload of the discipline "Numerical Methods" is 4 credit units.

Table 4.1. Types of educational work by periods of mastering the educational program of higher education for full-time education.

Type of academic work	**TOTAL,** academic hours		**Semester(s)**
			5
Contact work, academic hours	*54*		*54*
Lectures (LC)	18		18
Laboratory work (LW)	36		36
Practical/seminar classes (SC)	0		0
Independent work of students, academic hours	*90*		*90*
Control (exam/test with assessment), academic hours	*0*		*0*
Total complexity of the discipline	**academic hours**	***144***	*144*
	credit	***4***	*4*

The total workload of the discipline "Numerical Methods" is 4 credit units.

Table 4.2. Types of educational work by periods of mastering the educational program of higher education for full-time education.

Type of academic work	**TOTAL,** academic hours		**Semester(s)**
			5
Contact work, academic hours	*54*		*54*
Lectures (LC)	18		18
Laboratory work (LW)	36		36
Practical/seminar classes (SC)	0		0
Independent work of students, academic hours	*90*		*90*
Control (exam/test with assessment), academic hours	*0*		*0*
Total complexity of the discipline	**academic hours**	***144***	*144*
	credit	***4***	*4*

The total workload of the discipline “Numerical Methods” is 4 credit units.

Table 4.3. Types of educational work by periods of mastering the educational program of higher education for full-time education.

Type of academic work	**TOTAL,** academic hours		**Semester(s)**
			5
Contact work, academic hours	*54*		*54*
Lectures (LC)	18		18
Laboratory work (LW)	36		36
Practical/seminar classes (SC)	0		0
Independent work of students, academic hours	*90*		*90*
Control (exam/test with assessment), academic hours	*0*		*0*
Total complexity of the discipline	**academic hours**	***144***	*144*
	credit	***4***	*4*

5. CONTENT OF THE DISCIPLINE

Table 5.1. Content of the discipline (module) by types of academic work

Section number	Name of the discipline section	Topic Title		Topic Contents	Type of academic work*
Section 1	Numerical optimization methods	1.1	The concept of optimization	Definition of optimization as the process of finding the best (extreme) value of an objective function. Applications of optimization include engineering design, economics, management, and machine learning. Local and global optima.	LC, LW
		1.2	Statement of the optimization problem	Formal statement: objective function, decision variables, constraints (equalities and inequalities). Domain of feasible solutions. Problem classification: unconstrained and constrained optimization, one-dimensional and multidimensional, linear and nonlinear.	LC, LW
		1.3	Numerical approach to solving the optimization problem	Principles of numerical methods. Iterative nature: constructing a sequence of approximations to the optimal solution. Iterative termination criteria. Selecting the initial approximation.	LC, LW
Section 2	One-dimensional optimization methods	2.1	Swann's algorithm for finding the uncertainty interval	The problem of finding the interval containing the minimum of a unimodal function. Svenn's algorithm: moving with increasing steps until finding the point where the function begins to increase. Constructing an initial uncertainty interval.	LC, LW
		2.2	One-dimensional optimization methods	General characteristics of methods for finding the minimum of a function of one variable. Methods differ in the way they reduce the uncertainty interval. The requirement for the function to be unimodal.	LC, LW
		2.3	Bisection method	Sequentially divides the current interval into two equal parts. Calculates the function values at two interior points. Selects a new interval based on a comparison of these values. Convergence rate.	LC, LW
		2.4	Dichotomy method	A variation of the bisection method using two closely spaced points within an interval. Specifics of choosing the distance between the points. Advantages and disadvantages.	LC, LW
		2.5	Golden Ratio Method	Using the golden ratio to select interior points. The golden ratio property: one point is preserved when moving to a new interval. The method is optimal for unimodal functions.	LC, LW
		2.6	Fibonacci method	A method using Fibonacci numbers to determine the locations of sample points. The Fibonacci method is considered optimal among all sequential search methods for a fixed number of function evaluations.	LC, LW

Section number	Name of the discipline section	Topic Title		Topic Contents	Type of academic work*
Section 3	Multidimensional optimization methods	3.1	Multidimensional zero-order optimization methods	Methods that do not use derivatives of the target function. Applicability to nonsmooth or discontinuous functions, as well as when derivatives are difficult to calculate. These methods rely solely on calculating the function values.	LC, LW
		3.2	Hooke–Jeeves configuration method	A combination of exploratory search (coordinate-by-coordinate probing) and pattern-based search (movement toward a successful step). Robust and effective for many practical tasks.	LC, LW
		3.3	Nelder–Mead deformable polyhedron method	Construction of a simplex (polyhedron) in a space of variables. Operations: reflection, expansion, contraction, reduction. Adaptive reshaping of the simplex during the search for a minimum.	LC, LW
		3.4	Rosenbrock's method	Rotating coordinate method. Sequential execution of coordinate-by-coordinate steps followed by rotation of the coordinate system in the direction of steepest decay. Orthogonalization of search directions.	LC, LW
		3.5	Powell's conjugate direction method	Construction of a system of mutually conjugate search directions. Minimization is sequential along each direction, followed by updating the system of directions. Quadratic convergence.	LC, LW
		3.6	Random search methods	Using random directions and random steps. Advantages when the objective function has complex terrain and many local minima.	LC, LW
		3.7	Adaptive random search method	Automatically adjusts random search parameters (step size, success probability) during operation. Adaptation to local properties of the target function.	LC, LW
		3.8	Random search method with backtracking on failure	If a step fails (function value deteriorates), return to the previous point and generate a new random direction. Increasing search reliability.	LC, LW
		3.9	Best sample method	Generates a specified number of random sample points in the neighborhood of the current point. Selects the best of these as a new approximation. Controls the neighborhood size.	LC, LW
		3.10	Statistical gradient method	Estimating the gradient direction from a random sample of points. Using statistical estimates to predict the direction of decreasing function. Accounting for noise when calculating the function.	LC, LW
		3.11	Random search method with guiding hypersquare	Constructing a hypercube (hypersquare) around the current point. Generating sample points within the hypersquare. Adaptive resizing and reshaping of the hypersquare.	LC, LW
Section 4	Numerical methods of differentiation and integration	4.1	Numerical methods for approximate calculation of derivatives	The problem of approximately finding the derivative of a function given in a table or as a complex expression. Finite-difference ap-	LC, LW

Section number	Name of the discipline section	Topic Title		Topic Contents	Type of academic work*
				proximations. Order of accuracy. Effect of round-off errors.	
		4.2	Difference formula for calculating the first partial derivative	Approximation of the partial derivative of a function of several variables. Forward, backward, and central difference formulas. Selecting the discretization step.	LC, LW
		4.3	Difference formula for calculating the second derivative	Approximation of the second derivative using function values at three points. Central difference formula for the second derivative. Accuracy and stability.	LC, LW
		4.4	Numerical methods for solving ODEs	Classification of methods: single-step and multi-step, explicit and implicit. Application areas: modeling dynamic systems and physical processes.	LC, LW
		4.5	Cauchy problem	Statement of the Cauchy problem for a first-order ordinary differential equation: differential equation and initial condition. Existence and uniqueness of a solution.	LC, LW
		4.6	Numerical solution of the Cauchy problem	Principles of numerical integration of differential equations. Discretization of the independent variable. Grid construction. Approximate calculation of the solution at grid nodes.	LC, LW
		4.7	Euler's method	The simplest one-step method of first-order accuracy. The idea of linearizing the solution at each step. The formula for the explicit Euler method. The method's error and stability.	LC, LW
		4.8	Improved Euler methods	Higher-order accuracy methods. Euler's method with recalculation (predictor-corrector). Second-order Runge-Kutta methods (midpoint method, Heun's method). Accuracy is improved by additional calculations of the right-hand side.	LC, LW
Section 5	First-order optimization methods	5.1	First-order optimization methods	Methods that use information about the first derivatives (gradient) of the objective function. The gradient is the direction of steepest increase. The antigradient is the direction of steepest decrease.	LC, LW
		5.2	Constant-step gradient descent method	Movement from the current point in the direction of the antigradient with a fixed step size. The problem of step size selection: too small a step size leads to slow convergence, while too large a step size leads to divergence.	LC, LW
		5.3	Coordinate gradient descent method	Alternate minimization along each coordinate. At each step, movement along one coordinate axis in the direction of decreasing function. Simple to implement, but possible slow convergence.	LC, LW
		5.4	Steepest gradient descent method	At each step, the optimal step size in the direction of the antigradient is selected (one-dimensional minimization along the direction). Faster convergence compared to a constant step size.	LC, LW

Section number	Name of the discipline section	Topic Title		Topic Contents	Type of academi c work*
		5.5	Gauss–Seidel method	Sequential minimization for each coordinate using updated variable values. Features for quadratic functions. Relationship with the Gauss-Seidel method for systems of linear equations.	LC, LW
		5.6	Fletcher–Reeves method	Using conjugate directions to accelerate convergence. Updating the search direction based on the previous direction and the current gradient. Quadratic convergence for quadratic functions.	LC, LW
Section 6	Second-order optimization methods	6.1	Second-order optimization methods	Methods that utilize information about second derivatives (the Hessian matrix). Taking into account the curvature of the target function surface. Higher convergence rate (quadratic).	LC, LW
		6.2	Newton's method	Using a second-order expansion of the objective function. Newton step formula using the inverse Hessian matrix and gradient. Fast convergence near the minimum. Requirements for the Hessian matrix.	LC, LW
		6.3	Newton–Raphson method	The classical Newton method for solving systems of nonlinear equations in the context of optimization (searching for a stationary gradient point). Convergence conditions.	LC, LW
		6.4	Marquardt method	A combination of gradient descent and Newton's methods. Using a regularization parameter to control the step size. Stability away from the minimum and rapid convergence near the minimum.	LC, LW
Section 7	Constraint optimization methods	7.1	Penalty function methods in constrained optimization	Transforming a constrained problem into an unconstrained optimization problem. Adding a penalty for constraint violations to the objective function. Sequentially solving penalty problems.	LC, LW
		7.2	Penalty function method (external penalty method)	A penalty is assessed for any violation of constraints, even minor ones. The outer point may lie outside the allowed range. The penalty weight increases with each iteration.	LC, LW
		7.3	Barrier function method (internal penalty method)	A barrier function that tends to infinity as the boundary of the feasible region is approached. The requirement is that the feasible region must be strictly interior (there must be interior points). The barrier parameter is gradually reduced.	LC, LW
		7.4	Combined penalty function method	Combination of external and internal penalties. Applicability to problems with mixed constraints (equalities and inequalities). Flexible configuration.	LC, LW
Section 8	Linear programming problems	8.1	Statement of the linear programming problem	The objective function and constraints are linear. The feasible region is a convex polyhedron. The optimal solution is achieved at the vertex of the polyhedron. Examples: resource allocation problem, transportation problem.	LC, LW

Section number	Name of the discipline section	Topic Title		Topic Contents	Type of academic work*
		8.2	Canonical form of a linear programming problem and methods of reduction to it	Standard form: maximization or minimization, equality constraints, non-negativity of variables. Transforming inequalities into equalities using additional variables.	LC, LW
		8.3	Simplex method for solving linear programming problems	An iterative method for moving from one vertex of a polyhedron to an adjacent one while improving the objective function value. Selecting the resolving column and row. Rule for selecting the input and output variables. Optimality criterion.	LC, LW
		8.4	An algorithm for obtaining an admissible initial basis when solving a linear programming problem using the simplex method	The initial feasible vertex problem. The artificial basis method: introducing artificial variables. The two-stage simplex method. The M-method (large penalty method).	LC, LW
Section 9	Discrete optimization problems	9.1	Concept and class of discrete optimization problems	Variables take discrete (integer) values. Classes: integer programming, Boolean programming, combinatorial optimization. Computational complexity (NP-hardness).	LC, LW
		9.2	Classical discrete optimization problems	Traveling salesman problem (finding the shortest Hamiltonian cycle). Knapsack problem. Assignment problem. Set covering problem. Integer linear programming problem.	LC, LW
		9.3	Methods for solving discrete optimization problems	Exact methods: exhaustive search (practically inapplicable for large dimensions). Pruning methods. Branch and bound. Dynamic programming. Approximate and heuristic methods.	LC, LW
		9.4	Heuristic algorithms	Algorithms that provide a reasonably good solution in an acceptable amount of time without guaranteeing optimality. Greedy algorithms. Local search. Heuristics for specific problems.	LC, LW
		9.5	Branch and bound method	Recursive partitioning of the solution set into subsets (branching). Calculating estimates (bounds) for each subset. Pruning unpromising branches. Finding the exact solution.	LC, LW
		9.6	Dynamic programming method	Decomposing a problem into subproblems. Bellman's optimality principle. Recurrence relations. Applicability to problems with small state dimensions.	LC, LW
Section 10	Modern metaheuristic algorithms for global optimization	10.1	A class of metaheuristic algorithms for global optimization	Algorithms for finding the global optimum in multi-extremum problems. Using randomness and adaptation. Differences from classical local optimization methods.	LC, LW
		10.2	Evolutionary and population optimization methods	Methods based on the coevolution of a set of possible solutions (populations). Ideas from biological evolution: selection, heredity, variability.	LC, LW
		10.3	Evolutionary algorithms	General scheme: population initialization, fitness assessment, selection, variation (mutation and recombination), replacement.	LC, LW

Section number	Name of the discipline section	Topic Title		Topic Contents	Type of academic work*
				Classification: genetic algorithms, evolutionary strategies, genetic programming.	
		10.4	Genetic algorithm	Representation of solutions as chromosomes (fixed-length strings). Selection operators: roulette, tournament selection. Generation of a new generation.	LC, LW
		10.5	Crossover and mutation operations in genetic algorithm	Crossing over: the exchange of parts of the genetic code between two parental solutions. Mutation: a random change in one or more genes. The role of operators in exploring the search space.	LC, LW
		10.6	Population algorithms	Algorithms that use a population of agents interacting with each other and the environment. This differs from classical genetic algorithms.	LC, LW
		10.7	Particle swarm method	Simulating the behavior of a flock of birds or a school of fish. Each particle moves through the search space, remembering its best position and the best position of the swarm. Velocity and position are updated.	LC, LW
		10.8	Scheme of modification of a possible solution in the particle swarm method	Inertial component, cognitive component (memory of the best personal decision), social component (influence of the best global decision). Method parameters.	LC, LW
		10.9	Bee algorithm	Simulating the behavior of a bee colony searching for nectar sources. Scouts and foragers. Reconnaissance and exploitation stages. Local and global search mechanisms.	LC, LW
		10.10	Gray wolf algorithm	Simulation of the pack hierarchy and hunting mechanisms of gray wolves. The roles of alpha, beta, delta, and omega. Mechanisms for prey encirclement, pursuit, and attack.	LC, LW
		10.11	Cat optimization algorithm	Modeling two cat behavior modes: search mode (surveying the surroundings) and tracking mode (moving toward a target). A balance between exploration and exploitation.	LC, LW
		10.12	A method inspired by bats	An algorithm based on bat echolocation. Modifying the frequency, volume, and pulse rate. Adapting parameters during the search.	LC, LW
		10.13	Whale optimization algorithm	Imitation of humpback whale hunting behavior (the "bubble net" method). Mechanisms of prey encirclement, spiral movement, and random search.	LC, LW

* - to be completed only for FULL-TIME education: LC – lectures; LW – laboratory work; SC – practical/seminar classes.

6. LOGISTIC AND TECHNICAL SUPPORT OF DISCIPLINE

Table 6.1. Material and technical support for the discipline

Audience type	Equipment of the auditorium	Specialized educational/laboratory equipment, software and materials for mastering the discipline (if necessary)
Lecture	A lecture hall equipped with specialized furniture, a whiteboard (screen), and multimedia presentation equipment.	Projector
Computer class	A computer room for conducting classes, group and individual consultations, ongoing monitoring and midterm assessment, equipped with personal computers (14 in total), a board (screen) and technical means for multimedia presentations.	
Seminar	An auditorium for conducting seminar-type classes, group and individual consultations, ongoing monitoring and midterm assessment, equipped with a set of specialized furniture and technical means for multimedia presentations.	
For independent work	A classroom for independent student work (can be used for seminars and consultations), equipped with a set of specialized furniture and computers with access to the Electronic Information System.	

* - the classroom for independent work of students MUST be indicated!

7. EDUCATIONAL, METHODOLOGICAL AND INFORMATIONAL SUPPORT OF THE DISCIPLINE

Main literature:

1. Attetkov A.V., Galkin S.V., Zarubin V.S. Optimization Methods. Moscow: Publishing house of Bauman Moscow State Technical University. 2001. 440 p.
2. Panteleev A.V., Letova T.A. Optimization methods in examples and problems. Moscow: Higher School. 2002. 544 p.
3. Kornienko V.P. Optimization Methods. Moscow: Higher School. 2007. 664 p.
4. Sobol B.V., Meskhi B.Ch., Kanygin G.I. Optimization Methods. Workshop. Rostov-on-Don: Phoenix Publishing House. 2009. 380 p.

Further reading:

1. Gladkov L.A., Kureichik V.V., Kureichik V.M. Genetic algorithms: M.: Fizmatlit, 2006.- 319 p.
2. Chernorutsky I.G. Optimization methods in control theory
3. Izmailov A.F., Solodov M.V. Numerical methods of optimization
4. Andreeva E.A., Tsiruleva V.M. Variational calculus and optimization methods

Resources of the information and telecommunications network "Internet":

1. RUDN University Electronic Library System and third-party electronic library systems to which university students have access based on concluded agreements

- Electronic library system of RUDN - ELS RUDN http://lib.rudn.ru/MegaPro/Web
- Electronic Library System "University Library Online" http://www.biblioclub.ru
- EBS Yurayt http://www.biblio-online.ru
- Electronic Library System "Student Consultant" www.studentlibrary.ru
- Electronic Library System "Troitsky Bridge"

2. Databases and search engines
   - electronic fund of legal and regulatory documentation http://docs.cntd.ru/
   - Yandex search engine https://www.yandex.ru/
   - Google search engine https://www.google.ru/
   - SCOPUS abstract database http://www.elsevierscience.ru/products/scopus/

Educational and methodological materials for independent work of students in mastering a discipline/module:*

1. Lecture course on the subject "Numerical Methods".

* - all teaching and methodological materials for independent work of students are posted in accordance with the current procedure on the discipline page in TUIS!

DEVELOPERS:

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