Federal State Autonomous Educational Institution of Higher Education

PEOPLES' FRIENDSHIP UNIVERSITY OF RUSSIA
RUDN University

Faculty of Physics, Mathematics and Natural Sciences

educational division (faculty/institute/academy) as higher education programme developer

COURSE SYLLABUS

Topological methods in elliptic theory
course title

Recommended by the Didactic Council for the Education Field of:

01.04.01 Mathematics
field of studies / speciality code and title

The course instruction is implemented within the professional education programme of higher education:

«Functional methods in differential equations and interdisciplinary research»

higher education programme profile/specialisation title

2025

1. COURSE GOAL(s)

The discipline "Topological methods in elliptic theory" is one of the initial disciplines of the direction of preparation "Mathematics", from which the training in the magistracy begins and prepares the student for the study of subsequent disciplines.
The main goal of the course is to introduce students to the basic concepts and methods of elliptic theory, including the Fredholm theory and elements of algebraic topology. The development of these theories in the 20th century led to the famous Atiyah-Singer index formula for the index of an elliptic operator on a closed manifold

2. REQUIREMENTS FOR LEARNING OUTCOMES

Mastering the discipline " Topological methods in elliptic theory" is aimed at developing the following competencies (parts of competencies):

Table 2.1. The list of competencies formed by students in the course of mastering the discipline (the results of mastering the discipline)

Code	Competence	Competence achievement indicators (within this discipline)
GC-5	Able to analyze and take into account the diversity of cultures in the process of intercultural interaction	**GC-5.1.** Analyzes the most important ideological and value systems formed in the course of historical development; substantiates the relevance of their use in social and professional interaction; **GC-5.2.** Builds social and professional interaction taking into account the peculiarities of the main forms of scientific and religious consciousness, business and general culture of representatives of other ethnic groups and confessions, various social groups; **GC.5.3.** Ensures the creation of a non-discriminatory interaction environment when performing professional tasks
PC-10	Able to develop educational and methodological complexes for e-learning	**PC-10.1.** Ability to develop educational and methodological complexes for e-learning

3.COURSE IN HIGHER EDUCATION PROGRAMME STRUCTURE

The discipline "Topological methods in elliptic theory" refers to the part formed by the participants in the educational relations of block B1 of the EP HE.

As part of the EP HE, students also master other disciplines and / or practices that contribute to the achievement of the planned results of mastering the discipline " Topological methods in elliptic theory ".

Table 3.1. The list of the higher education programme components/disciplines that contribute to the achievement of the expected learning outcomes as the course study results

Code	Competence	Previous disciplines/modules, practices	Subsequent disciplines/modules, practices*
GC-5	Able to analyze and take into account the diversity of cultures in the process of intercultural interaction	-	State examination
PC-10	Able to develop educational and methodological complexes for e-learning	-	State examination

4. COURSE WORKLOAD AND ACADEMIC ACTIVITIES

The total labor intensity of the discipline " Topological methods in elliptic theory " is 4 credits.

*Table 4.1. Types of academic activities during the periods of higher education programme mastering (**full-time training**)**

Type of study work		TOTAL, a.h.	Semester			
			1	2	3	4
Contact work, academic hours		*72*		*72*		
Lectures (LC)		36		36		
Lab work (LW)						
Seminars (workshops/tutorials) (S)		36		36		
Self-studies		*36*		*36*		
Evaluation and assessment (exam/passing/failing grade)		*36*		*36*		
Course workload	a.h.	**144**		**144**		
	credits	**4**		**4**		

5. COURSE CONTENTS

Table 5.1. Course contents and academic activities types

Course Module Title	Brief Description of the Module Content	Type of study work
Section 1. Theory of Fredholm	Compact operators. Finite rank operators. compact operators. *-Algebra of compact operators. Compactness conditions for integral operators. Fredholm operators. Definition of Fredholm property. Properties of Fredholm operators. Nikolsky-Atkinson theorem. Index definition. Index properties. Toeplitz operators Boundedness. Symbol. composition formula. Theorem on the Fredholm property of Toeplitz operators.	Lecture, seminar

Section 2. Elements of algebraic topology	The fundamental group. Paths. Loops. Path homotopy. Definition of fundamental group. Induced mapping. Its homotopy invariance. Fundamental group of space II Homotopy equivalences of spaces. Invariance of the fundamental group under homotopy equivalences. Coverings. Universal coverings. Calculation of the fundamental group with the help of coverings. Differential forms and de Rham cohomology	Lecture, seminar
Section 3. Elements of index theory of elliptic operators	Formulas for the index of Toeplitz operators. Differential operators. Ellipticity. Pseudodifferential operators. Formulas for the Index of Elliptic Operators in Rn.	Lecture, seminar

6. CLASSROOM EQUIPMENT AND TECHNOLOGY SUPPORT REQUIREMENTS

Table 6.1. Classroom equipment and technology support requirements

Classroom type	**Classroom equipment**	**Specialized educational/laboratory equipment, software and materials for mastering the discipline**
Lecture	An auditorium for lecture-type classes, equipped with a set of specialized furniture; board (screen) and technical means of multimedia presentations.	-
Seminar	An auditorium for conducting seminar-type classes, group and individual consultations, current control and intermediate certification, equipped with a set of specialized furniture and technical means for multimedia presentations.	-
For independent work of students	An auditorium for conducting seminar-type classes, group and individual consultations, current control and intermediate certification, equipped with a set of specialized furniture and technical means for multimedia presentations.	-

7. RESOURCES RECOMMENDED FOR COURSE STUDY

Main literature:

1. Fomenko, A. T. and Mishchenko, A. S. A short course in differential geometry and topology. Cambridge Scientific Publishers, Cambridge. 2009.
2. Fomenko A.T., Mishchenko A.S., Solovyev Yu P. Selected Problems in Differential Geometry and Topology. Cambridge Scientific Publishers UK. 2013
3. Palais, Richard S. Seminar on the Atiyah-Singer index theorem, Annals of Math. Studies, No. 57, Princeton University Press, Princeton, N.J. 1965

Additional literature:

1. Zhang, Weiping, Lectures on Chern-Weil theory and Witten deformations, World Scientific Publishing Co., Inc., River Edge, NJ}, 2001
2. Solovyov, Yu. P. and Troitsky, E. V. C*-algebras and elliptic operators in differential topology, American Mathematical Society, Providence, RI}, 2001

Resources of the information and telecommunications network "Internet":

1. RUDN ELS and third-party ELS, to which university students have access on the basis of concluded agreements:
- RUDN Electronic Library System - RUDN EBS http://lib.rudn.ru/MegaPro/Web
- ELS "University Library Online" http://www.biblioclub.ru
- EBS Yurayt http://www.biblio-online.ru
- ELS "Student Consultant" www.studentlibrary.ru
- EBS "Lan" http://e.lanbook.com/
- EBS "Trinity Bridge"
- video recordings of lectures by A.Yu. Savin: https://teach-in.ru/course/elliptic-operators-and-index-theorem (in Russian)
- lecture notes of the lectures by A. Yu. Savin (in Russian): https://teach-in.ru/file/synopsis/pdf/elliptic-operators-and-index-theorem-M.pdf
- 35 video lectures on index theory by Masoud Khalkhali: https://youtube.com/playlist?list=PLFLLJhtIPhRY11aAbO4N8KYAwGrOQMg9Q

2. Databases and search engines:
- electronic fund of legal and normative-technical documentation http://docs.cntd.ru/
- Yandex search engine https://www.yandex.ru/
- Google search engine https://www.google.ru/
- abstract database SCOPUS http://www.elsevierscience.ru/products/scopus/

8. ASSESSMENT TOOLKIT AND GRADING SYSTEM* FOR EVALUATION OF STUDENTS' COMPETENCES LEVEL UPON COURSE COMPLETION

Evaluation materials and a point-rating system* for evaluating the level of formation of competencies (parts of competencies) based on the results of mastering this discipline are presented in the Appendix to this Work Program of the discipline

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