Federal State Autonomous Educational Institution of Higher Education
Peoples' Friendship University of Russia named after Patrice Lumumba

Academy of Engineering
(name of the main educational unit (MEU) that developed the educational program of higher education)

WORKING PROGRAM OF THE DISCIPLINE

NUMERICAL METHODS FOR SOLVING MATHEMATICAL MODELING PROBLEMS

(name of discipline/module)

Recommended for the field of study/specialty:

27.04.04 CONTROL IN TECHNICAL SYSTEMS
(code and name of the field of study/specialty)

The discipline is mastered within the framework of the implementation of the main professional educational program of higher education (EP HE):

Artificial Intelligence, Machine Learning, and Space Science
(name (profile/specialization) of the educational institution of higher education)

2026

1. THE GOAL OF MASTERING THE DISCIPLINE

The course "Numerical Methods for Solving Mathematical Modeling Problems" is part of the master's program "Artificial Intelligence, Machine Learning, and Space Sciences" in the 27.04.04 "Control in Technical Systems" program and is studied in the first semester of the first year. The course is offered by the Department of Mechanics and Control Processes. It consists of 6 sections and 45 topics and focuses on the theory and practical application of computer-based research methods and solutions for extreme value problems. Specific iterative direct and indirect numerical optimization methods are studied.

The objective of this course is to provide students with the necessary foundational knowledge in the fundamental methods of numerically solving optimization problems for single- and multivariable functions, numerical optimization methods for convex functions, and numerical methods for solving problems of variational calculus and optimal control. This knowledge should also be used rationally and effectively when implementing the corresponding algorithms on a computer. This course also provides students with an understanding of the appropriate method to choose in a specific situation, depending on the problem statement. The main objectives of the course are: creating favorable conditions for student self-development; introducing students to the basic concepts of modern mathematics; and developing skills in numerically solving optimization problems.

2. REQUIREMENTS FOR THE RESULTS OF MASTERING THE DISCIPLINE

Mastering the course "Numerical Methods for Solving Mathematical Modeling Problems" aimed at developing the following competencies (parts of competencies) in students:

Table 2.1. List of competencies developed in students while mastering the discipline (results of mastering the discipline)

Cipher	**Competence**	**Indicators of Competency Achievement** (within this discipline)
GPC-1	Able to analyze and identify the natural scientific essence of control problems in technical systems based on provisions, laws and methods in the field of natural sciences and mathematics	GPC-1.1 Knows the basic laws, provisions and methods in the field of natural sciences and mathematics; GPC-1.2 Able to identify the natural scientific essence of control problems in technical systems guided by the laws and methods of natural sciences and mathematics; GPC-1.3 Proficient in tools for analyzing control problems in technical systems.
GPC-2	Able to formulate control problems in technical systems and justify methods for solving them	GPC-2.1 Knows the basic methods of solving control problems in technical systems; GPC-2.2 Able to justify methods for solving control problems in technical systems; GPC-2.3 Proficient in methods of setting control problems in technical systems.
GPC-8	Able to select methods and develop control systems for complex technical objects and technological processes	GPC-8.1 Knows the basic methods used to develop control systems for complex technical objects and technological processes; GPC-8.2 Able to develop control systems for complex technical objects and technological processes; GPC-8.3 Has the skills to select methods and develop control systems for complex technical objects and technological processes.

3. PLACE OF THE DISCIPLINE IN THE STRUCTURE OF THE EDUCATIONAL EDUCATIONAL INSTITUTION

Course "Numerical Methods for Solving Mathematical Modeling Problems" refers to the mandatory part of block 1 “Disciplines (modules)” of the educational program of higher education.

As part of the higher education program, students also master other disciplines and/or practices that contribute to the achievement of the planned results of mastering the discipline "Numerical Methods for Solving Mathematical Modeling Problems."

Table 3.1. List of components of the educational program of higher education that contribute to the achievement of the planned results of mastering the discipline

Cipher	**Name of competence**	**Previous courses/modules, practical training***	**Subsequent disciplines/modules, practices***
GPC-1	Able to analyze and identify the natural scientific essence of control problems in technical systems based on provisions, laws and methods in the field of natural sciences and mathematics		Undergraduate Training; Advanced Methods of Space Flight Mechanics; Advanced Methods of Earth Remote Sensing; Geoinformation Systems and Applications;
GPC-2	Able to formulate control problems in technical systems and justify methods for solving them		Dynamics and Control of Space Systems; Undergraduate Training;
GPC-8	Able to select methods and develop control systems for complex technical objects and technological processes		Undergraduate Training;

* - filled in accordance with the competency matrix and the SUP EP HE
** - elective courses/practices

4. SCOPE OF THE DISCIPLINE AND TYPES OF EDUCATIONAL WORK

The total workload of the course “Numerical methods for solving problems of mathematical modeling” is 5 credit units.

Table 4.1. Types of educational work by periods of mastering the educational program of higher education for full-time education.

Type of academic work	**TOTAL,** academic hours		**Semester(s)**
			1
Contact work, academic hours	*34*		*34*
Lectures (LC)	17		17
Laboratory work (LW)	0		0
Practical/seminar classes (SC)	17		17
Independent work of students, academic hours	*110*		*110*
Control (exam/test with assessment), academic hours	*36*		*36*
Total complexity of the discipline	**academic hours**	***180***	*180*
	credit	***5***	*5*

5. CONTENT OF THE DISCIPLINE

Table 5.1. Content of the discipline (module) by types of academic work

Section number	Name of the discipline section	Topic Title		Topic Contents	Type of academic work*
Section 1	Methods for minimizing functions of one variable	1.1	Problem statement. Classical method	Definition of the problem of minimizing a function of one variable as finding the point at which the function attains its minimum value. Description of the classical method: finding the derivative of the function, equating it to zero to determine stationary points, and checking the sign of the second derivative to identify the minimum.	LC, SC
		1.2	Bisection method	Definition of the bisection method as a method of successively dividing a segment containing a minimum in half. Description of the algorithm: selecting two interior points on the segment, calculating the function values at these points, comparing the results, and shortening the segment on one side. Characteristic of the linear rate of convergence of the method.	LC, SC
		1.3	Golden Ratio Method	Definition of the golden ratio method as a method for finding a minimum using the golden ratio. Description of the algorithm: dividing a segment by the golden ratio, sequentially eliminating parts of the segment where the minimum is clearly absent. Characterizing the method as optimal among sequential search methods.	LC, SC
		1.4	Broken line method	Definition of the broken line method as a global optimization method for functions with bounded second derivatives. Description of the construction of a piecewise linear broken line that bounds the original function from below. Characterization of the successive refinement of the lower bound for the function and finding the global minimum.	LC, SC
		1.5	Coating method	Definition of the covering method as a global minimization method by uniformly enumerating points on a given segment. Algorithm description: constructing a grid with a specified step, calculating function values at the grid nodes, and selecting the smallest value. Characterization of the dependence of the result's accuracy on the number of points and the step length.	LC, SC
		1.6	Convex functions of one variable	A convex function is defined as a function whose interval between any two points on the graph lies no lower than the graph itself. A	LC, SC

Section number	Name of the discipline section	Topic Title		Topic Contents	Type of academic work*
				characteristic property of convex functions is that any local minimum is also a global minimum. A description of the convexity criterion is given in terms of the sign of the second derivative.	
		1.7	Method of tangents	Definition of the tangent method as a method for minimizing convex functions using the derivative. Algorithm description: constructing tangents to the function graph at current points and finding the intersection point of the tangents with the horizontal axis. Characteristic of the method's high quadratic convergence rate.	LC, SC
Section 2	Classical theory of extremum of functions of several variables	2.1	Statement of the problem	Definition of the problem of minimizing a function of many variables as finding a set of numerical values for which the objective function achieves its smallest possible value. Characterization of the domain as part of a multidimensional space. Description of local and global extrema.	LC, SC
		2.2	Weierstrass's theorem	Statement of the Weierstrass theorem: a continuous function on a closed and bounded set necessarily attains its largest and smallest values. Description of the conditions for the applicability of the theorem. Characterization of its consequences for optimization problems.	LC, SC
		2.3	Classical method for solving problems on unconditional extremum	Description of the classical method: equating all partial derivatives of the objective function to zero to find stationary points. Characteristics of the use of the matrix of second partial derivatives. Description of criteria for verifying sufficient conditions for an extremum.	LC, SC
		2.4	Problems on conditional extremum	Definition of a conditional extremum problem as the minimization of a function subject to equality constraints. Description of the variable elimination method. Formulation of the Lagrange multiplier method: introduction of an auxiliary function that includes the original function and constraints, followed by a search for its stationary points.	LC, SC
		2.5	Necessary conditions of the first and second order	Description of first-order necessary conditions: equality to zero of all partial derivatives of the auxiliary Lagrange function at the extremum point. Characteristic of second-order necessary conditions: definite sign of the matrix of second derivatives of the auxiliary function.	LC, SC
		2.6	Sufficient conditions for an extremum	Description of sufficient conditions for an extremum: positive definiteness of the matrix of second derivatives for the minimum	LC, SC

Section number	Name of the discipline section	Topic Title		Topic Contents	Type of academic work*
				and negative definiteness for the maximum. Characteristics of verification of Legendre conditions. Description of sufficient conditions for problems with and without constraints.	
Section 3	Methods for minimizing functions of several variables	3.1	Gradient method	The definition of the gradient descent method is as a descent method in the direction of the function's steepest decay, i.e., in the direction opposite to the gradient. Description of the iterative process: successive movement from the current point to a new one in the direction of the antigradient. Characteristics of the step length selection: constant step, fractional step, or steepest descent.	LC, SC
		3.2	Gradient projection method	Definition of the gradient projection method for problems with constraints. Algorithm description: calculate the antigradient, take a step in that direction, and then project the resulting point back onto the feasible region. Application characteristics for convex feasible sets.	LC, SC
		3.3	Conditional gradient method	Definition of the conditional gradient method as the Frank-Wolfe method for problems with linear constraints. Algorithm description: linearize the objective function at the current point, solve a linear programming problem to find the descent direction, and perform a step along the found direction.	LC, SC
		3.4	Method of possible directions	Definition of the feasible directions method as a constraint-based descent method. Algorithm description: search for a direction that forms an acute angle with the antigradient while remaining within the feasible region. Characterization of the Zoitendijk conditions for constructing directions.	LC, SC
		3.5	Proximal method	The proximal method is defined as a method with a quadratic addition that penalizes the distance from the previous point. Description of the iterative process: at each step, an auxiliary problem is solved: minimizing the sum of the objective function and the square of the distance to the previous point. Characterization of the method's robustness to objective function nonsmoothness.	LC, SC
		3.6	Linearization method	Definition of the linearization method as a method for replacing a nonlinear problem with a sequence of linear problems. Algorithm description: linearization of the objective function and constraints at the current point, solving the resulting linear problem to determine the next approximation. Convergence characteristics and applicability of the method.	LC, SC

Section number	Name of the discipline section	Topic Title		Topic Contents	Type of academic work*
		3.7	Quadratic programming	Definition of quadratic programming as the problem of minimizing a quadratic objective function under linear constraints. Description of solution methods: simplex method for problems with quadratic functions, active constraint method. Characterization of the Karsh-Kuhn-Tucker optimality conditions for quadratic programming.	LC, SC
		3.8	Conjugate Directions Method	The conjugate direction method is defined as a method that uses a system of directions conjugate with respect to the matrix of second derivatives. Algorithm description: sequential descent along conjugate directions, which ensures finding the minimum of a quadratic function in a finite number of steps. Characteristics of the method's high efficiency.	LC, SC
		3.9	Newton's method	Newton's method is defined as a second-order method that uses both first and second derivatives of the objective function. Algorithm description: constructing a quadratic approximation of the function at the current point and finding its minimum. It exhibits quadratic convergence, but is computationally expensive due to the calculation and inversion of the matrix of second derivatives.	LC, SC
		3.10	Continuous methods with variable metric	Definition of variable-metric methods as methods that approximate the inverse of the second derivative matrix, avoiding its direct computation. Description of the Davidson-Fletcher-Powell and Broyden-Fletcher-Goldfarb-Shanno algorithms. Characterization of the combination of the high convergence rate of the Newton method and the moderate computational complexity of gradient methods.	LC, SC
		3.11	Coordinate descent method	The coordinate descent method is defined as a method that changes only one coordinate of the vector of variables at each step. Algorithm description: Alternate minimization of the function along each coordinate axis. Characterized by ease of implementation and low convergence rate, especially near the minimum.	LC, SC
		3.12	Covering method in multidimensional problems	Definition of the covering method as a global optimization method using uniform enumeration of points in a multidimensional domain. Algorithm description: constructing a multidimensional grid, calculating function values at the nodes, and selecting the best point. Characteristic of exponential growth of the number of points with increasing problem dimension.	LC, SC
		3.13	Modified Lagrange function method	Definition of the modified Lagrange function method as a method	LC, SC

Section number	Name of the discipline section	Topic Title		Topic Contents	Type of academic work*
				for solving problems with constraints by adding quadratic penalty terms to the Lagrange function. Algorithm description: sequential solution of auxiliary problems with refinement of the Lagrange multipliers and penalty parameter. Characteristics of overcoming the problem of unlimited growth of the penalty parameter.	
		3.14	Penalty function method	Definition of the penalty function method as a method for reducing a constrained problem to a sequence of unconstrained problems. Algorithm description: adding a penalty term to the objective function that increases when the constraints are violated. Characterization of the need to increase the penalty parameter to infinity to ensure exact constraint satisfaction.	LC, SC
		3.15	Proof of necessary conditions for first- and second-order extrema using penalty functions	A description of the use of the penalty function method to prove necessary conditions for an extremum. The approach is characterized by constructing a penalty problem that approximates the original problem and analyzing the optimality conditions for the penalty problem, followed by limiting. A description of how to obtain first-order conditions in the form of the Lagrange function gradient being equal to zero.	LC, SC
		3.16	Barrier function method	Definition of the barrier function method as a method for solving problems with inequality constraints. Algorithm description: adding a barrier term that approaches infinity as it approaches the boundary of the feasible region. Application characteristics for problems where points on the boundary are inadmissible. Description of logarithmic and inverse barriers.	LC, SC
		3.17	Loaded function method	Definition of the loaded function method as a global optimization method that uses modification of the objective function to overcome local minima. Algorithm description: adding special additives to the original function that change the function's relief in the region of identified local minima. Application characteristics for finding a global minimum.	LC, SC
		3.18	Random search method	Definition of the random search method as a method using random directions and random steps to explore the search space. Description of algorithms: random search with backtracking, best-trial, adaptive random search. Performance characteristics for multidimensional problems with ravine terrain and discontinuous functions.	LC, SC

Section number	Name of the discipline section	Topic Title		Topic Contents	Type of academic work*
Section 4	Dynamic programming	4.1	Bellman's diagram	Definition of dynamic programming as a method for solving multi-step optimization problems. Formulation of Bellman's optimality principle: an optimal policy has the property that, whatever the initial state and decision, subsequent decisions must constitute an optimal policy with respect to the state that arises after the first decision. Description of Bellman's recurrence equation.	LC, SC
		4.2	The problem of synthesis for discrete systems	Definition of the synthesis problem as constructing control as a function of the current system state. Description of the solution to the synthesis problem for discrete systems using dynamic programming. Characteristic of the time-inverse approach for calculating the Bellman function and subsequently constructing control.	LC, SC
		4.3	Moiseev's scheme	Definition of the Moiseev scheme as a dynamic programming variant for continuous-time control optimization problems. Description of the approach: discretization of the problem in time and reduction to a multi-step process. Characteristics of the application of the Moiseev scheme in computational practice.	LC, SC
		4.4	Synthesis problem for continuous-time systems	Description of the control synthesis problem for continuous systems. Definition of the Bellman function as a solution to the Hamilton-Jacobi-Bellman equation. Characterization of the difficulties in solving this equation in multidimensional problems. Description of the relationship between the optimality principle and Pontryagin's maximum principle.	LC, SC
		4.5	Sufficient conditions for optimality	Formulation of sufficient conditions for optimality in dynamic programming. Description of the condition on the Bellman function: if a function is found that satisfies the Hamilton-Jacobi-Bellman equation with boundary conditions, then the control constructed based on it is optimal. Characterization of the role of sufficient conditions in verifying the optimality of the resulting solution.	LC, SC
Section 5	Pontryagin's maximum principle	5.1	Statement of the optimal control problem	An optimal control problem is defined as the problem of finding a control that transitions a system from its initial state to its final state while minimizing a given performance functional. Description of the problem components: phase variables, control variables, differential constraints, boundary conditions, and objective functional.	LC, SC
		5.2	Formulation of the maximum principle	Formulation of Pontryagin's maximum principle: for optimal con-	LC, SC

Section number	Name of the discipline section	Topic Title		Topic Contents	Type of academic work*
				trol, there exists a nonzero vector function of adjoint variables satisfying the adjoint system such that at each instant of time, the control maximizes the Hamiltonian function. Description of the Hamiltonian function and the adjoint system of differential equations.	
		5.3	Proof of the maximum principle	General characteristics of the proof of the maximum principle. Description of the control variation method. Characterization of the needle variation idea: local control variation over a small interval. Description of obtaining optimality conditions in the form of the maximum of the Hamiltonian function and the fulfillment of transversality at the endpoints of the trajectory.	LC, SC
		5.4	Maximum principle for optimal control problems with phase constraints	Definition of a problem with phase constraints as a problem where the trajectory must not leave a given region of phase space. Description of a modification of the maximum principle for such problems. Characteristics of the appearance of additional conditions at the boundary of phase constraints. Description of adjacency conditions and contact conditions.	LC, SC
		5.5	The relationship between the maximum principle and classical calculus of variations	A description of the relationship between the maximum principle and the classical calculus of variations. A characterization of the Euler-Lagrange equations as a consequence of the maximum principle in the absence of control constraints. A description of the Weierstrass condition for a strong minimum. A characterization of the maximum principle as a generalization of classical conditions to the case of bounded controls.	LC, SC
Section 6	Application of the maximum principle to optimization problems	6.1	Reducing an optimization problem to a boundary value problem of the maximum principle	Description of the procedure for reducing an optimal control problem to a boundary value problem. Characterization of the stages: constructing the Hamiltonian function, writing out the adjoint system, finding the control from the Hamiltonian maximum condition, substituting the control into the phase and adjoint systems. Description of the derivation of a two-point boundary value problem with boundary conditions specified partially at the initial and partially at the final time.	LC, SC
		6.2	Shooting method for numerical solution of the boundary value problem of the maximum principle	Definition of the shooting method as an iterative method for solving boundary value problems. Algorithm description: specifying missing initial conditions for the adjoint variables, integrating the system from the initial to the final instant, calculating residuals of	LC, SC

Section number	Name of the discipline section	Topic Title		Topic Contents	Type of academic work*
				the final conditions, and adjusting the initial conditions using Newton's method. Characterizing the sensitivity of the method to the initial approximation.	
		6.3	Modifications of Newton's method: Isaev-Sonin modification, Fedorenko normalization	Description of a modification of the Newton-Isaev-Sonin method for solving boundary value problems based on the maximum principle. Characteristic of the feature: use of a special procedure for recalculating the matrix of derivatives. Description of Fedorenko normalization as a method for eliminating the degeneracy of a boundary value problem by adding additional equations. Characteristic of increasing the stability of the calculations.	LC, SC
		6.4	Runge-Kutta method for solving Cauchy problems	Definition of the Runge-Kutta method as a numerical method for integrating ordinary differential equations. Description of the operating principle: calculating the function increment per step as a weighted sum of the right-hand side values at several points within the step. Characterization of the fourth-order accuracy method as the most common. Description of the application of the Runge-Kutta method to integrating phase and conjugate systems in the shooting method.	LC, SC

* - to be completed only for FULL-TIME education: LC – lectures; LW – laboratory work; SC – practical/seminar classes.

6. LOGISTIC AND TECHNICAL SUPPORT OF DISCIPLINE

Table 6.1. Material and technical support for the discipline

Audience type	Equipment of the auditorium	Specialized educational/laboratory equipment, software and materials for mastering the discipline (if necessary)
Lecture	A lecture hall equipped with specialized furniture, a whiteboard (screen), and multimedia presentation equipment.	
Seminar	An auditorium for conducting seminar-type classes, group and individual consultations, ongoing monitoring and midterm assessment, equipped with a set of specialized furniture and technical means for multimedia presentations.	
For independent work	A classroom for independent student work (can be used for seminars and consultations), equipped with a set of specialized furniture and computers with access to the Electronic Information System.	

* - the classroom for independent work of students MUST be indicated!

7. EDUCATIONAL, METHODOLOGICAL AND INFORMATIONAL SUPPORT OF THE DISCIPLINE

Main literature:

1. Bakhvalov Nikolay Sergeevich. Numerical methods: Textbook / N.S. Bakhvalov, N.P. Zhidkov, G.M. Kobelkov; N.S. Bakhvalov and others - 4th ed. - M.: Nauka, 1987. - 636 p. : ill. - (Classical university textbook). - ISBN 5-94774-396-5: 244.53.

2. Kalitkin Nikolay Nikolaevich. Numerical Methods: A Textbook for Universities / N.N. Kalitkin; Ed. by A.A. Samarsky. - M.: Nauka, 1978. - 512 p.: ill. - 1.30.

3. Rozova Valentina Nikolaevna. Optimization Methods: Lecture Course: Tutorial / V.N. Rozova, I.S. Maksimova. - M.: RUDN University, 2010. - 109 p. - ISBN 978-5-209-038-72-6

4. Vasiliev F.P. Numerical methods for solving extremal problems. Moscow, Nauka, 1988 - 549 p.

5. Vasiliev F. P. Optimization Methods. Moscow: Factorial Press, 2002 - 524 p.

6. Alekseev V.M., Galeev E.M., Tikhomirov V.M. Collection of optimization problems: Theory. Examples. Tasks. - M.: Nauka, 1984. - 288 p.

7. Alekseev V.M., Tikhomirov V.M., Fomin S.V. Optimal control. Moscow, Nauka. 1979. - 429 p.

8. Galeev E.M., Tikhomirov V.M. A Short Course in the Theory of Extremal Problems. Moscow: Moscow State University Press, 1989. - 203 p.

9. Pontryagin L.S., Boltyansky V.G., Gamkrelidze R.V., Mishchenko E.F. Mathematical theory of optimal processes. Moscow, Nauka, 1969 - 384 p.

Further reading:

1. Fedorenko R.P. Approximate solutions of optimal control problems. Moscow, Nauka, 1978.

2. A. N. Kolmogorov, S. V. Fomin. Elements of the Theory of Functions and Functional

Analysis. Lomonosov Moscow State University. - 7th ed. - Moscow: Fizmatlit, 2004. - 572 p.

3. Grigoriev K.G., Grigoriev I.S., Zapletin M.P. Practical training in numerical methods in optimal control problems. Supplement 1, Moscow, Publishing House of the Center for Applied Research at the Faculty of Mechanics and Mathematics of Moscow State University, 2007.

4. Grigoriev I.S. Methodological manual on numerical methods for solving boundary value problems of the maximum principle in optimal control problems, Moscow, Publishing House of the Center for Applied Research at the Faculty of Mechanics and Mathematics of Moscow State University, 2005

5. Bakhvalov N.S., Zhidkov N.P., Kobelkov G.M. Numerical methods, M., BINOM. Knowledge Laboratory, 2008.

6. Filippov A. F. Collection of problems in differential equations. - Izhevsk: Research Center "Regular and Chaotic Dynamics", 2000, 176 p.

7. Gabasov R., Kirillova F.M. Special optimal controls. – Moscow: Nauka, 1973. – 256 p.

Resources of the information and telecommunications network "Internet"

Resources of the information and telecommunications network "Internet":

1. RUDN University Electronic Library System and third-party electronic library systems to which university students have access based on concluded agreements
   - Electronic library system of RUDN - ELS RUDN http://lib.rudn.ru/MegaPro/Web
   - Electronic Library System "University Library Online" http://www.biblioclub.ru
   - EBS Yurayt http://www.biblio-online.ru
   - Electronic Library System "Student Consultant" www.studentlibrary.ru
   - Electronic Library System "Troitsky Bridge"
2. Databases and search engines
   - electronic fund of legal and regulatory documentation http://docs.cntd.ru/
   - Yandex search engine https://www.yandex.ru/
   - Google search engine https://www.google.ru/
   - SCOPUS abstract database http://www.elsevierscience.ru/products/scopus/

Educational and methodological materials for independent work of students in mastering a discipline/module:*

1. Lecture course on the subject "Numerical methods for solving mathematical modeling problems".

* - all teaching and methodological materials for independent work of students are posted in accordance with the current procedure on the discipline page in TUIS!

DEVELOPERS:

Associate Professor		Samokhin Alexander Sergeevich
Position, DEPARTMENT	*Signature*	*Surname I.O.*
Associate Professor		Saltykova Olga Alexandrovna
Position, DEPARTMENT	*Signature*	*Last name I.O.*

HEAD OF THE DEPART-MENT:

Head of Department		Razumny Yuri Nikolaevich
Position of the DEPARTMENT	*Signature*	*Surname I.O.*

HEAD OF THE EP HE:

Professor		Razumny Yuri Nikolaevich
Position, DEPARTMENT	*Signature*	*Surname I.O.*